1. Shown in the figure below is the view from the front of a sliding bar of resistance \( R \) and length \( L \). The bar is free to slide \emph{vertically} along the rails without friction. The system is immersed in a magnetic field, \( B \), pointed out of the page. Gravity will cause the bar to begin sliding down the rails. Answer all the following:

(a) Write an expression for the flux through the loop formed by the bar and wire slide for the moment when the bar’s position is “\( y \)”.

\[ \Phi_B = B L y \] (1)

(b) Using your result from (a) determine an expression for the voltage generated in the loop as a function of the bar’s velocity.

\[ \frac{d\Phi_B}{dt} = \frac{d(BL y)}{dt} = BL \frac{dy}{dt} = BL v \] (2)

\[ V = -N \frac{d\Phi_B}{dt} = -BL v \] (3)

(full credit even w/o minus)

(c) Determine the current through the bar.

\[ I = \frac{V}{R} = -\frac{BL v}{R} \] (4)

(d) Determine the \emph{magnetic} force on the bar as a function of velocity (\emph{i.e.} ignore gravity during this step).

\[ F = ILB = -\frac{BL}{R} v \times LB = -\frac{B^2 L^2}{R} v \] (5)

(e) What is the terminal velocity of the bar?

\[ \frac{B^2 L^2}{R} v_T = mg \] (6)

\[ v_T = \frac{mgR}{B^2 L^2} \] (7)
2. The circuit below begins with the switch in the open position. The battery has $V_B = 8\text{V}$, the resistor has $R = 2\,\text{k}\Omega$, and the inductor has $L = 3\,\text{mH}$. At time=0, the switch in the circuit is closed.

(a) Determine the time constant of this circuit.

$$
\tau = \frac{L}{R} = \frac{3\,\text{mH}}{2\,\text{k}\Omega} = 1.5\,\mu\text{sec}
$$

(b) Draw sketches of the time dependence of each of the following:

i. The voltage on the inductor as a function of time, $V_L(t)$.

![Graph 1](image1.png)

ii. The voltage on the resistor as a function of time, $V_R(t)$.

![Graph 2](image2.png)

iii. The current through the resistor as a function of time, $I_R(t)$.

![Graph 3](image3.png)

To be counted for full credit the vertical axis of each sketch should be labelled with a **numerical value** indicating either the initial or asymptotic value of the quantity plotted.

(c) Write an equation for each of the following:
i. The voltage on the inductor as a function of time, $V_L(t)$,

$$V_L(t) = 8Ve^{-\frac{t}{\tau_L}}$$  \hfill (9)

ii. The voltage on the resistor as a function of time, $V_R(t)$,

$$V_R(t) = 8V \left(1 - e^{-\frac{t}{\tau_R}}\right)$$  \hfill (10)

iii. The current through the resistor as a function of time, $I_R(t)$,

$$I_R(t) = 4mA \left(1 - e^{-\frac{t}{\tau_R}}\right)$$  \hfill (11)

(d) At what time is the voltage on the inductor 3 V?

$$3V = 8Ve^{-\frac{t}{\tau_L}}$$  \hfill (12)

$$0.375 = e^{-\frac{t}{\tau_L}}$$  \hfill (13)

$$\ln 0.375 = -\frac{t}{1.5\mu sec}$$  \hfill (14)

$$t = -1.5\mu sec \times \ln 0.375 = 1.47\mu sec$$  \hfill (15)
3. Shown below is an RC circuit driven by an AC power source.

(a) Draw a phasor diagram representing this circuit.

(b) Determine the magnitude of the impedance, $|\bar{Z}|$.

$$|\bar{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$  \hspace{1cm} (16)

(c) Determine the phase of the impedance, $\phi_Z$.

$$\phi_Z = \tan^{-1}\left(\frac{-1}{\omega CR}\right) = \tan^{-1}\left(\frac{-1}{R^2C}\right)$$  \hspace{1cm} (17)

(d) Explain in a single sentence what the phase of the impedance means.
   The phase of the impedance equals the phase difference between the total voltage and total current.

(e) Let the resistance be $R = 1.0k\Omega$, the capacitance be $C = 0.5\mu F$ and the voltage source have an RMS voltage, $V_{RMS} = 12V$, at a frequency of $f = 300Hz$. Determine the RMS current through the circuit.

$$|\bar{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$  \hspace{1cm} (18)

$$\frac{1}{2\pi f C} = \frac{1}{2\pi \times 300 \times 0.5 \times 10^{-6}} = 1061\Omega$$  \hspace{1cm} (19)

$$|\bar{Z}| = \sqrt{1000^2 + 1061^2} = 1458\Omega$$  \hspace{1cm} (20)

$$I_{RMS} = \frac{V_{RMS}}{|\bar{Z}|} = \frac{12}{1458} = 8.23mA$$  \hspace{1cm} (21)

(f) Determine the RMS voltage across the capacitor.

$$V_{RMS(cap)} = I_{RMS} |\bar{Z}|_{cap}$$  \hspace{1cm} (22)

$$V_{RMS(cap)} = I_{RMS} \frac{1}{2\pi f C}$$  \hspace{1cm} (23)

$$V_{RMS(cap)} = 8.23mA \times 1061\Omega = 8.73V$$  \hspace{1cm} (24)
(g) What circuit element would you add to this circuit (i.e. making a new circuit) such that the result was a resonant circuit with resonant frequency $f = 30kHz$? Please specify the type of circuit element (resistor, capacitor, or inductor) and the value (resistance, capacitance, or inductance) required.

Add an inductor such that:

$$\omega = \frac{1}{\sqrt{LC}} \quad (25)$$

$$\sqrt{L} = \frac{1}{\omega \sqrt{C}} \quad (26)$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C} = 56.3\mu H \quad (27)$$
4. Shown in the figure below is a system containing an object, and two lenses. Use the shapes of the lenses in the figure to decide whether they are converging or diverging optical elements.

(a) Find the image location and magnification of the first lens (assuming that only this lens exists). Specify this image location, \( d_{i1} \), as some number of centimeters to the left or to the right of this lens.

\[
d_{o1} = 15\text{cm} \quad f_1 = 8\text{cm} \quad (28)
\]

\[
\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \quad (29)
\]

\[
d_d = 17.14\text{cm} \quad (30)
\]

\[
m_1 = -\frac{17.14\text{cm}}{15\text{cm}} = -1.143 \quad (31)
\]

(right of lens)

(b) The image of the first lens acts as the object for the second. Find the location and magnification of the image produced by the second lens. Specify this image location, \( d_{o2} \), as some number of centimeters to the left or to the right of this lens.

\[
d_{o2} = -13.14\text{cm} \quad f_2 = -30\text{cm} \quad (32)
\]

\[
\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \quad (33)
\]

\[
d_{i2} = 23.38\text{cm} \quad (34)
\]

\[
m_1 = -\frac{23.38\text{cm}}{-13.14\text{cm}} = 1.779 \quad (35)
\]

(right of lens)

(c) Calculate the total magnification of this entire system.

\[
m_{TOT} = m_1 \times m_2 = -2.034 \quad (36)
\]

(d) Is the final image real or virtual?

real
5. Shown in the figure below is Young’s two-slit experiment. Monochromatic light of wavelength \( \lambda = 0.633 \mu m \) comes in from the left and passes through the two (very narrow) slits. The result is a pattern of bright and dark bands visible on a screen far away from the two slits. **WARNING**: this drawing is not to scale.

(a) Determine the distance, \( d \), between the two slits,

\[
\tan \theta_1 = \frac{0.08}{12} \quad (37)
\]
\[
\theta_1 = 0.381^\circ \quad (38)
\]
\[
d \sin \theta_1 = \lambda \quad (39)
\]
\[
d \sin 0.381^\circ = 0.633 \mu m \quad (40)
\]
\[
d = 95 \mu m \quad (41)
\]

(b) If the light is changed to green light, \( \lambda = 0.532 \mu m \), the diffraction pattern will change. What is the location, \( y \), of the first dark band using this light.

\[
d \sin \theta_\frac{\lambda}{2} = \frac{\lambda}{2} \quad (42)
\]
\[
95 \mu m \sin \theta_\frac{\lambda}{2} = \frac{0.532 \mu m}{2} \quad (43)
\]
\[
\theta_\frac{\lambda}{2} = 0.161^\circ \quad (44)
\]
\[
y = 12 m \tan \theta_\frac{\lambda}{2} = 12 m \tan 0.161^\circ = 3.37 cm \quad (45)
\]

(c) Make a sketch of the diffraction pattern that would result if the double-slit used for the problem so far was replaced by a single slit.