Physics 122
Midterm Examination #1
March 21, 2007

SOLUTIONS
1. Shown in the figure below is a rectangular loop located to the left of a region with a magnetic field \( B = 0.25 \text{ Tesla} \) pointed out of the paper. The loop is dragged with a constant speed of \( v = 3 \text{ m/s} \) along the path shown by the dashed arrow. The resistance of the loop is \( R = 8 \Omega \).

(a) Determine the magnitude \textbf{AND direction} (clockwise or counter-clockwise) of the current in the loop during each of the following three conditions:

i. The loop is entering the field region.

\[
\Phi_B = Blx \quad (1)
\]
\[
\frac{d\Phi_B}{dt} = Bl \frac{dx}{dt} = Blv \quad (2)
\]
\[
I = \frac{Blv}{R} = \frac{0.25 \cdot 0.2 \cdot 3}{8} = 0.01875 \text{ A} = 18.75 \text{ mA} \quad (4)
\]

\textit{clockwise} \quad (5)

ii. The loop is completely inside the field region.

\[
I = 0 \quad (6)
\]

iii. The loop is leaving the field region.

\[
\Phi_B = Blx \quad (7)
\]
\[
\frac{d\Phi_B}{dt} = Bl \frac{dx}{dt} = Blv \quad (8)
\]
\[
V = Blv \quad (9)
\]
\[ I = \frac{Blv}{R} = \frac{0.25 \cdot 0.3 \cdot 3}{8} = 0.0281 \text{ A} = 28.1 \text{ mA} \quad (10) \]

\[ \text{counter-clockwise} \quad (11) \]

(b) Determine the magnitude \textbf{AND} direction of the force on the loop as it is exiting the field. Indicate the direction of the force by circling one of the following:

i. Toward the left of the page.

ii. \textbf{Toward the top of the page}.

iii. Toward the right of the page.

iv. Toward the bottom of the page.

v. Out of the page.

vi. Into the page.

\[ F = IlB = 0.0281 \cdot 0.3 \cdot 0.25 = 2.1 \times 10^{-3} \text{ N} \quad (12) \]
2. Shown below is an LRC circuit.

(a) Draw a phaser diagram representing this circuit.

(b) Analyze your phaser diagram to determine the magnitude of the impedance, $|Z|$.

$$|Z_{tot}| = \sqrt{R^2 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2} \quad (13)$$

(c) Analyze your phaser diagram to determine the phase of the impedance, $\phi_Z$.

$$\phi_Z = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \quad (14)$$

(d) Let the resistance be $R = 600 \, \Omega$, the inductance be $L = 1.04 \, \mu H$ and the capacitance be $C = 3 \times 10^{-12} \, F$. Determine the peak voltage across the resistor if the circuit is driven by an AC power source with $V_{peak} = 10 \, V$ and $f = 500 \, MHz$. 

\[ |Z_{\text{tot}}| = \sqrt{R^2 + \left( \frac{\omega L}{\omega C} - \frac{1}{\omega C} \right)^2} \] \hspace{1cm} (15)

\[ \omega L = 2\pi 500 \times 10^6 \left( 1.04 \times 10^{-6} \right) = 3267 \Omega \] \hspace{1cm} (16)

\[ \frac{1}{\omega C} = \frac{1}{2\pi 500 \times 10^6 3 \times 10^{-12}} = 106 \Omega \] \hspace{1cm} (17)

\[ |Z_{\text{tot}}| = \sqrt{600^2 + (3267 - 106)^2} \] \hspace{1cm} (18)

\[ |Z_{\text{tot}}| = 3217 \Omega \] \hspace{1cm} (19)

\[ V_R = \frac{\omega}{Z_{\text{tot}}} V_p = \frac{600}{3217} 10 = 1.87 \text{ V} \] \hspace{1cm} (20)

(e) Clearly, your circuit is not tuned for the \( f = 500 \text{ MHz} \) signal. Determine the frequency that your circuit is tuned for.

\[ \omega = \frac{1}{\sqrt{LC}} \] \hspace{1cm} (21)

\[ \omega = \frac{1}{\sqrt{1.04 \times 10^{-6} 3 \times 10^{-12}}} = 566 \times 10^6 \] \hspace{1cm} (22)

\[ f = \frac{\omega}{2\pi} = 90.1 \times 10^6 \text{ Hz} = 90.1 \text{ MHz} \] \hspace{1cm} (23)

(f) **EXTRA CREDIT** (1 point): What are the call letters of this radio station?

WUSB
3. Two parallel conductors carry currents in opposite directions, as shown in the Figure below. One conductor carries a current of \(10.0\) A. Point A is the midpoint between the wires, and point C is \(5.00\) cm to the right of the \(10.0\) A current. I is adjusted so that the magnetic field at C is zero.

(a) Find the value of the current I.

\[
0 = \frac{\mu_0 I}{2\pi 0.15} - \frac{\mu_0 10}{2\pi 0.05}
\]

\[
\frac{\mu_0 I}{2\pi 0.15} = \frac{\mu_0 10}{2\pi 0.05}
\]

\[
I \frac{1}{0.15} = \frac{10}{0.05}
\]

\[
I = 30\text{ Amps}
\]

(b) Find the value of the magnetic field at A.

\[
B = \frac{\mu_0 30}{2\pi 0.05} + \frac{\mu_0 10}{2\pi 0.05}
\]

\[
B = \frac{\mu_0 40}{2\pi 0.05}
\]

\[
B = 0.00016\text{ Tesla}
\]
4. The circuit below shows a generator driving a transformer. The output side of the transformer has an AC voltage whose RMS is $V_{RMS} = 110 \, V$ and whose frequency is $f = 60 \, Hz$.

The generator is made from a circular coil of wire with $N = 400$ turns whose radius is $r = 0.2 \, m$. The coil is rotated in a uniform magnetic field, $B$.

(a) Determine the peak voltage applied to the input side of the transformer.

$$V_{in}(RMS) = \frac{3000}{100} \times 110 = 3300 \, Volts$$

$$V_{in}(peak) = \sqrt{2} V_{in}(RMS) = 4667 \, Volts$$

(b) Determine the magnetic field in the generator.

$$V_{peak} = \omega N B A$$

$$B = \frac{V_{peak}}{\omega N A} = \frac{V_{peak}}{2\pi f N \pi r^2}$$

$$B = \frac{4667}{2\pi \times 60 \times (400) \pi (0.2)^2} = 0.246 \, Tesla$$
5. Shown in the figure below is a toroid. The toroid carries a current $I$ and has a total of $N$ turns. Use Ampere’s Law to calculate the magnetic field at the radius $r$ shown in the Figure.

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I_{in}$$  \hspace{1cm} (36)$$

$$BL = \mu_0 I_{in}$$ \hspace{1cm} (37)$$

$$L = 2\pi r$$ \hspace{1cm} (38)$$

$$I_{in} = NI$$ \hspace{1cm} (39)$$

$$B = \frac{\mu_0 NI}{2\pi r}$$ \hspace{1cm} (40)$$

**HINT:** Think of a toroid as a long solenoid bent into a circular shape.