1. Hooray! You’ve finished your physics final exam and decide to teach your $10 calculator what projectile motion feels like by throwing it off the top of a tall building as shown below:

(a) (10 pts) What is the maximum height reached by the calculator?

\[
y(v_y = 0) =? \\
v_y^2 = v_{0y}^2 - 2g(y - y_0) \\
0 = v_{0y}^2 - 19.6(y - 20) \\
v_{0y} = v_0 \times \sin 30 = 5 \\
v_{0x} = v_0 \times \cos 30 = 8.66 \\
0 = 25 - 19.6(y - 20) \\
y = 25/19.6 + 20 = 21.27m
\]

(b) (10 pts) What is the direction, \( \theta \), of the velocity vector that the calculator has just before it hits the ground?

\[
v_x(y = 0) = v_{0x} = 8.66 \\
v_y(y = 0) =? \\
v_y^2 = v_{0y}^2 - 2g(y - y_0) \\
v_y^2 = 25 - 19.6(0 - 20) = 417 \\
v_y = -20.42 \\
\theta = \arctan \frac{v_y}{v_x} = \arctan \frac{-20.42}{8.66} = -67^\circ
\]

**NOTE:** Both positive and negative values for \( \theta \) in part b) will be accepted for full credit.
2. Shown in the figure below are two blocks connected by a string. The two blocks are made of different materials and have different coefficients of kinetic friction (\(\mu_k = 0.2\) for the bottom block and \(\mu_k = 0.4\) for the top block).

![Diagram of two blocks connected by a string with forces acting on them.](image)

(a) (10 pts) What is the acceleration of the system?
(b) (10 pts) What is the tension in the cord?

Both normal forces are the same:

\[
N_1 = N_2 = 2g \cos \theta = 18.42N \tag{14}
\]

\[
2g \sin 20 + T - 0.4(18.42) = 2a \tag{15}
\]

\[
2g \sin 20 - T - 0.2(18.42) = 2a \tag{16}
\]

\[
4g \sin 20 - 0.6(18.42) = 4a \tag{17}
\]

\[
a = \frac{4g \sin 20 - 0.6(18.42)}{4} = 0.5888 \frac{m}{s} \tag{18}
\]

\[
T = 2a + 0.4(18.42) - 2g \sin 20 = 1.842N \tag{19}
\]
3. Shown in the figure below are two identical blocks of mass 1.5 kg. This first block is pressed against a spring \( k = 2400 \frac{N}{m} \) compressing it a distance \( x = 0.1 \ m \). The second mass is attached to the end of a string of length \( L = 2.2 \ m \). The system is released from rest.

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\[ \begin{align*}
x = 0.1 \ m \\
k = 2400 \ \text{N/m} \\
1.5 \ \text{kg} \\
\end{align*} \]
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(a) \((5 \ \text{pts})\) What is the velocity of the first block just before it collides with the second one?

\[
\frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 \tag{20}
\]

\[
\frac{1}{2} 2400(0.1)^2 = \frac{1}{2} 1.5 v_2^2 \tag{21}
\]

\[
v_2 = 4 \frac{m}{s} \tag{22}
\]

(b) \((5 \ \text{pts})\) What is the velocity of the pair of blocks just after they collide? \textbf{(Note:} the two blocks stick together after colliding). \(m v_2 = (m + m)v_3\) \(v_3 = \frac{1.5}{1.5 + 1.5} = \frac{2m}{s} \)

\[
\begin{align*}
\frac{1}{2}(m + m)v_3^2 &= (m + m)gy_4 \\
y_4 &= \frac{v_3^2}{2g} = 0.204m
\end{align*} \tag{26}
\]

(d) \((5 \ \text{pts})\) What is the period of the pendulum?
\[ \omega = \sqrt{\frac{g}{T}} = \sqrt{\frac{9.8}{2.2}} = 2.11 \]  \hspace{1cm} (28)

\[ T = \frac{2\pi}{\omega} = 2.98 \text{sec} \]  \hspace{1cm} (29)
4. In a famous James Bond movie, the imposter British agent is spotted by ordering red wine with fish! Worse still (but not shown in the movie), this same imposter ordered a Martini with an ice cube instead of an olive!

After drinking the martini, the imposter swirled the ice cube (uniform circular motion) in a circle of radius $R = 0.02 \, m$, as shown in the figure below. The mass of the ice cube is $m = 0.02 \, kg$.

(a) (6 pts) Find the normal force of the martini glass on the ice cube.
\begin{equation}
N \cos 60 - mg = 0 \tag{30}
\end{equation}
\begin{equation}
N = \frac{0.02(9.8)}{\cos 60} = 0.392N \tag{31}
\end{equation}

(b) (8 pts) Find the linear velocity, \(v\), of the ice cube.

\begin{equation}
N \sin 60 = m \frac{v^2}{R} \tag{32}
\end{equation}
\begin{equation}
v^2 = \frac{RN \sin 60}{m} = 0.3395 \tag{33}
\end{equation}
\begin{equation}
v = 0.583 \frac{m}{s} \tag{34}
\end{equation}

(c) (6 pts) Find the angular velocity, \(\omega\), of the ice cube in \(\text{rad/s}\).

\begin{equation}
\omega = \frac{v}{r} = \frac{.583}{0.02} = 29.13 \frac{\text{rad}}{s} \tag{35}
\end{equation}

**NOTE:** Friction is negligible in this problem.
5. Steam burns are said to be worse than water burns. Let’s find out how much worse. The table below contains some constants that may or may not be useful.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{\text{ice}} )</td>
<td>( \frac{2050}{\text{kg} \cdot \text{K}} )</td>
</tr>
<tr>
<td>( c_{\text{water}} )</td>
<td>( \frac{4186}{\text{kg} \cdot \text{K}} )</td>
</tr>
<tr>
<td>( c_{\text{steam}} )</td>
<td>( \frac{2080}{\text{kg} \cdot \text{K}} )</td>
</tr>
<tr>
<td>( L_{\text{fusion}} )</td>
<td>( \frac{334000}{\text{J}} )</td>
</tr>
<tr>
<td>( L_{\text{vaporization}} )</td>
<td>( \frac{2258000}{\text{J}} )</td>
</tr>
<tr>
<td>( T_{\text{melt}} )</td>
<td>( 0^\circ \text{C} )</td>
</tr>
<tr>
<td>( T_{\text{boil}} )</td>
<td>( 100^\circ \text{C} )</td>
</tr>
</tbody>
</table>

(a) (7 pts) Calculate the amount of heat, \( Q \), absorbed by your skin if it is put in contact with 10 grams of water at \( (T_i = 100^\circ \text{C}) \).

\[
Q = mc\Delta T = 0.01(4186) \times (100 - 37) = 2637J
\]  

(36)

(b) (7 pts) Calculate the amount of heat, \( Q \), absorbed by your skin if it is put in contact with 10 grams of steam at \( (T_i = 100^\circ \text{C}) \).

\[
Q = mc\Delta T + mL_f = 2637 + 0.01 \times 2258000 = 25217J
\]  

(37)

(c) (6 pts) What mass of water at \( T_i = 100^\circ \text{C} \) puts the same amount of heat into your skin as the steam in part b.

\[
mc\Delta T = 25217J
\]  

(38)

\[
m(4186)(100 - 37) = 25217
\]  

(39)

\[
m = \frac{25217}{4186 \times (100 - 37)} = 0.0956 \text{kg}
\]  

(40)

NOTE: The final temperature in all of the above is skin temperature of \( T_f = 37^\circ \text{C} \).
6. You set out upon the water in a simple rectangular boat with mass \( m = 3000 \text{ kg} \), length \( L = 3 \text{ m} \), and width \( w = 2 \text{ m} \) as shown in the figure below.

![Rectangular Boat Diagram](image)

(a) (7 pts) Calculate the depth of the bottom of the boat, \( y \), when it floats at equilibrium.

\[
\rho_f [2 \times 3 \times y] g = mg \tag{41}
\]
\[
\rho_f [2 \times 3 \times y] = m \tag{42}
\]
\[
y = \frac{m}{\rho_f [2 \times 3]} = 0.5m \tag{43}
\]

(b) (7 pts) **OH NO!!!**. Your boat has a leak in the bottom. Calculate the velocity of the water coming through the hole.

\[
P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \tag{44}
\]
\[
0 + 0 + 1000(9.8)0.5 = 0 + \frac{1}{2} (1000) v_2^2 + 0 \tag{45}
\]
\[
(9.8)0.5 = \frac{1}{2} v_2^2 \tag{46}
\]
\[
v_2 = \sqrt{2(9.8) \times 0.5} = 3.13 \frac{m}{s} \tag{47}
\]

(c) (6 pts) If the hole is circular with radius \( r = 0.005 \text{ m} \), calculate the volume rate of flow \( \left( \frac{m^3}{s} \right) \) of water entering your boat.

\[
Flow = Av = \pi r^2 v = \pi (0.005)^2 3.13 = 2.46 \times 10^{-4} \frac{m^3}{s} \tag{48}
\]
7. Shown in the picture below is a saxophone.

(a) (5 pts) Circle the sketch the represents the Node/Anti-node pattern when the saxophone is played in the low register in the fundamental mode. Label your circle with the letter “A”.

(b) (5 pts) What length of the cavity, \( L \), would produce a note with frequency \( f = 116.5 \text{ Hz} \) (a B-flat).

\[
\begin{align*}
f_1 &= \frac{1}{4L} v \\ 116.5 &= \frac{345}{4L} \\ L &= \frac{345}{4(116.5)} = 0.74 m
\end{align*}
\] (49) (50) (51)

(c) (5 pts) Circle the part of the saxophone that opens to make the saxophone play one octave higher (twice the previous frequency).

(d) (5 pts) Circle the node pattern when the saxophone is played as in part c). Label this circle as “D”.
8. 2 moles of Neon, $N_e$, gas are taken through the cycle shown in the figure below ($a \rightarrow b \rightarrow c$). At point $a$, the temperature is 300 K and the pressure is 300,000 Pa. During the process $a \rightarrow b$, the volume of the system doubles.

![Diagram of a cycle with $P\rightarrow V$ axes showing points $a$, $b$, and $c$. An arrow indicates the isothermal process.]

(a) (9 pts) Calculate and fill in the “state table” below:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>V</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>30000</td>
<td>0.03326</td>
<td>300</td>
</tr>
<tr>
<td>b</td>
<td>30000</td>
<td>0.03326</td>
<td>600</td>
</tr>
<tr>
<td>c</td>
<td>15000</td>
<td>0.03326</td>
<td>300</td>
</tr>
</tbody>
</table>

(b) (12 pts) Calculate and fill in the “process table” below:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
<th>$\Delta U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a $\rightarrow$ b</td>
<td>-4989</td>
<td>12472.5</td>
<td>7483.5</td>
</tr>
<tr>
<td>b $\rightarrow$ c</td>
<td>0</td>
<td>-7483.5</td>
<td>-7483.5</td>
</tr>
<tr>
<td>c $\rightarrow$ a</td>
<td>3458</td>
<td>-3458</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>-1531</td>
<td>1531</td>
<td>0</td>
</tr>
</tbody>
</table>

(c) (4 pts) Calculate the heat from the hot reservoir, $Q_H$, and the heat to the cold reservoir, $Q_C$.

\[
Q_H = 12472.5 J \quad (52)
\]

\[
Q_C = -10941.5 J \quad (53)
\]

(d) (5 pts) Calculate the efficiency of your engine.

\[
\epsilon = \left| \frac{W}{Q_H} \right| = 0.123 \quad (54)
\]