1. The Super Soaker water gun was a revolution in water gun technology. The pump handle is moved back and forth building up significant pressure in the water reservoir. The water is released by pulling a trigger and shoots for a significant distance. Assume that one can make an absolute pressure of 8 atmospheres in the reservoir. Find all the following:

(a) What is the velocity at which the water leaves the gun?

**ANSWER:** Well, this is a Bernoulli equation problem. The Bernoulli equation is used to connect two locations within a single fluid. We will choose our two locations as being at the surface of the water in the reservoir (location 1) and at the exit of the gun. This means that:

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]  

At location 1, we approximate the water velocity is zero. Location 1 has an absolute pressure of 8 atm or 800,000 Pa. Location 2 touches the air and has pressure 1 atm = 100,000 Pa:

\[ 800000 + 0 + 1000 \times 9.8 \times 0.08 = 100000 + \frac{1}{2} \rho v_2^2 + 0 \]  

\[ 700784 = \frac{1}{2} 1000v_2^2 \]  

\[ 700784 = 500v_2^2 \]  

\[ v_2^2 = 1402 \]  

\[ v_2 = 37.44 \text{ m/s} \]

(b) If the water exits the gun through a hole with radius 1 mm, what is the volume rate of flow \( \frac{m^3}{s} \)?

**ANSWER:** The volume rate of flow at any location is the cross sectional area multiplied by the velocity:

\[ \text{Flow}_{volume} = Av \]
\[ \text{Flow}_{\text{volume}} = \pi r^2 v \]  
(8)

\[ \text{Flow}_{\text{volume}} = \pi 0.0005^2 37.44 \]  
(9)

\[ \text{Flow}_{\text{volume}} = 0.000029 \frac{m^3}{s} \]  
(10)

(c) If the water gun is fired horizontally and held 1.2 meters above the ground, where does the water hit the ground?

**Answer:** AH HA!! A projectile problem. We would like to know:

\[ x(y = 0) =? \]  
(11)

Well, the only variable that the x and y equations have in common is \( t \)! So first we find when the water hits the ground:

\[ t(y = 0) =? \]  
(12)

\[ y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]  
(13)

\[ 0 = 1.2 + 0 - 4.9t^2 \]  
(14)

\[ t^2 = \frac{-1.2}{-4.9} \]  
(15)

\[ t = 0.49 \text{ sec} \]  
(16)

Now finally we solve the x position at this time:

\[ x(t = 0.49) =? \]  
(17)

\[ x = x_0 + v_{0x} t \]  
(18)

\[ x = 0 + 37.44 \times 0.49 \]  
(19)

\[ x = 18.5 \text{ meters} \]  
(20)

**Note:** The reservoir is a “tank” of water in which the water moves with negligible velocity.
2. You are given a rectangular Cu bar whose length is 80 cm and whose cross section is 1 cm-by-1 cm. You perform many experiments on this very same Cu bar. Answer all of the following (NOTE: You will need to consult many tables from your book to get necessary constants).

(a) What is the mass of the bar?
(b) You apply equal but opposite forces (F=500 Newtons) to the ends of the bar in an attempt to stretch the bar. What is the new length of the bar?
(c) You put the bar in a vacuum and it swells a tiny bit. What is the change in volume of the bar?
(d) You apply a shear stress to the bar of $10^7$ Pa. The forces are applies to the “long sides” of the bar (the 80x1 cm$^2$ faces). What is the deformation of the bar $(x)$?
(e) How much force (applied as in part b) would be required to break the bar?
(f) What is the apparent weight of the bar when it is under water?
(g) If you raise the temperature of the bar from 20° C to 30° C, what is the change in its length?
(h) Suppose that during the previous temperature rise, the bar were FORCED to remain at the original length. What is the “thermal stress” on the bar?
(i) If you raise the temperature of the bar from 20° C to 30° C, what is the change in its volume?
(j) How much heat was required to raise the temperature of the bar from 20° C to 30° C?
(k) How much heat is required to take your Cu bar from room temp (20° C) to its melting point?
(l) How much heat is required to melt the bar (starting already at the temperature equal to the melting point)?
(m) You place one end of your bar in ice water and the other end in boiling water. What is the rate at which heat flows through the bar?
(n) Suppose your bar is inside a vacuum box and has a temperature of 500° C.
   i. What is the rate at which heat flows off the bar into the vacuum (ignore any heat flows into the bar)?
   ii. Assuming that the walls of the vacuum box are at 20° C, what is the rate at which heat enters the bar from the vacuum box?
   iii. What is the net rate of heat leaving the bar?

   NOTE: Assume an emissivity of $e = 1$ throughout this problem.
3. A collection of nitrogen gas molecules is at $P = 100,000 \, \text{Pa}$, $T = 25^\circ \, \text{C}$, and is held in a box of Volume $V = 2 \, \text{m}^3$. Answer all the following:

(a) How many molecules are in the box?
(b) What is the average Translational Kinetic Energy of one of these molecules?
(c) What is the average velocity of one of these molecules?
(d) What is the internal energy of the system?
(e) How much heat is required to raise the temperature of the gas to $35^\circ \, \text{C}$ if the volume is not changed as the heat is added?
(f) How much heat is required to raise the temperature of the gas to $35^\circ \, \text{C}$ if the volume is allowed to change in such a way that the pressure is not changed?
(g) What is the change in internal energy of the gas during each of the two previous processes?
4. A warm summer day has a pressure of 100,000 Pa and a temperature of 300 K.

(a) Find the number of moles per cubic meter of any ideal gas under these conditions.

**ANSWER:**

\[
\frac{n}{V} = \frac{P}{RT} = \frac{100000}{8.315 \times 300} = 40.1 \text{ moles/m}^3
\]

(b) Find the number of \( \frac{kg}{m^3} \) of Nitrogen gas, \( N_2 \), under these conditions.

**ANSWER:** \( N_2 \) has 28 grams per mole

\[
40.1 \frac{\text{moles}}{m^3} \times \frac{0.028 \text{kg}}{\text{mole}} = 1.12 \frac{\text{kg}}{m^3}
\]

(c) Find the number of \( \frac{kg}{m^3} \) of Helium gas, \( He \), under these conditions.

**ANSWER:** \( He \) has 4 grams per mole

\[
40.1 \frac{\text{moles}}{m^3} \times \frac{0.004 \text{kg}}{\text{mole}} = 0.160 \frac{\text{kg}}{m^3}
\]

(d) Determine the net force of a 1.5 \( m^3 \) Helium balloon in air (take the calculation for \( N_2 \) as a good approximation of air).

**ANSWER:** The Helium experiences a gravity force down and a Bouyant force up:

\[
F_{\text{tot}} = F_B - mg
\]

\[
F_{\text{tot}} = \rho_{N_2} V g - \rho_{He} V g
\]

\[
F_{\text{tot}} = (\rho_{N_2} - \rho_{He}) V g
\]

\[
F_{\text{tot}} = (1.12 - 0.16) 1.5 (9.8) = 14 N
\]

(e) Determine the RMS velocity of the He atoms.

**ANSWER:** Here we use the relationship between temperature and average kinetic energy:

\[
< KE_{tr} > = \frac{3}{2} kT
\]

\[
\frac{1}{2} m_{He} < v^2 > = \frac{3}{2} kT
\]

\[
\sqrt{< v^2 >} = \sqrt{\frac{3kT}{m_{He}}}
\]
\[
\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3(N_A k) T}{m_{He} N_A}}
\]  
(32)

\[
\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M_{He}}}
\]  
(33)

\[
\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3(8.315) 300}{0.004}} = 1368 \frac{m}{s}
\]  
(34)
5. A 0.100 kg Cu cup contains 0.800 kg of water at 20° C. 0.020 kg of ice whose temperature is −10° C is dropped into the cup. Determine the final temperature of the system.

**ANSWER:** The key to solving this problem is to set the total heat flow to zero. The total heat flow contains some positive and some negative terms:

\[
Q = 0 = m_Cu c_Cu (T_f - 20) + m_w c_w (T_f - 20) + m_i c_i (0 - -10) + m_i L_f + m_i c_w (T_f - 0) \quad (35)
\]

\[
0 = T_f (m_Cu c_Cu + m_w c_w + m_i c_i + m_i L_f - 20 (m_Cu c_Cu + m_w c_w)) \quad (36)
\]

\[
0 = T_f (0.1(390) + 0.8(4186) + 0.02(2100)) + 0.02(2100)10 + 0.02(333000) - 20(0.1(390) + 0.8(4186)) \quad (37)
\]

\[
0 = T_f (3429.8) - 60676 \quad (38)
\]

\[
T_f = 17.7° C \quad (39)
\]
6. You travel in a spaceship to Mars. We will approximate your ship as a sphere of radius 10 meters and take its emissivity as 1.0.

(a) Assuming the surface of your ship is at the same 290 K as the interior, what is the rate at which the ship loses heat via radiation (in Watts)?

**Answer:** We use the Steffan-Boltzmann law to determine the rate of heat loss due to radiation:

\[ H = e\sigma AT^4 \]  \hspace{1cm} (40)

\[ H = 1.0(5.67 \times 10^{-8})4\pi r^2290^4 \]  \hspace{1cm} (41)

\[ H = 1.0(5.67 \times 10^{-8})4\pi 10^2290^4 = 504000W \]  \hspace{1cm} (42)

(b) The sun has a radius of 695,000,000 m and a surface temperature of roughly 5,800 K. How many \( \frac{W}{m^2} \) are emitted from the sun’s surface by radiation?

**Answer:** We use the Steffan-Boltzmann law to determine the rate of heat loss due to radiation:

\[ H = e\sigma AT^4 \]  \hspace{1cm} (44)

\[ H/A = e\sigma T^4 \]  \hspace{1cm} (45)

\[ H/A = 1.0(5.67 \times 10^{-8})5800^4 = 6400000W/m^2 \]  \hspace{1cm} (46)

(c) Fortunately for you, you are much further than only 695,000 km from the center of the sun. You are 150 billion m from its center. How many \( \frac{W}{m^2} \) is the sunlight intensity at your ship?

**Answer:** We know that intensity is inversely proportional to radius squared

\[ I \propto \frac{1}{r^2} \]  \hspace{1cm} (47)

\[ Ir^2 = \text{const} \]  \hspace{1cm} (48)

\[ I_1r_1^2 = I_2r_2^2 \]  \hspace{1cm} (49)

\[ I_2 = \frac{r_2^2}{r_1^2}I_1 \]  \hspace{1cm} (50)

\[ I_2 = \frac{695000000^2}{150000000000^2} \times 64000000 = 1377 \frac{W}{m^2} \]  \hspace{1cm} (51)

(d) How many Watts do you receive from the sun?

**Answer:** The cross sectional area of your ship is \( \pi r^2 = 314 \ m^2 \).

\[ \text{Power from Sun} = 1377 \times 314 = 432744W/m^2 \]  \hspace{1cm} (52)

(e) Do you worry most about frying or freezing on your trip to Mars?

You would worry most about freezing since you are losing energy...however the sun does make up for about 90% of the energy you would otherwise lose!!! Makes sense since this is how earth is heated.
7. You have in your possession two rectangular bars. Each of these bars is 30 cm in length and has a square cross section of 2 cm by 2 cm. The first bar is made of steel \( k = 40 \frac{W}{m \cdot K} \) and the second is made of Aluminum \( k = 200 \frac{W}{m \cdot K} \). You decide to do two experiments in heat transfer as shown in the figure below.

\[
\begin{array}{lll}
T = 100 & T = 0 & T = 100 & T = 0 \\
\hline
\text{Experiment A} & \text{Experiment B}
\end{array}
\]

(a) Calculate the total heat transfer rate, \( H \), for experiment A.

**ANSWER:** This is the easy case since the two bars have known temperatures at their two ends. First for the steel bar:

\[
H_1 = k \frac{A}{L} \Delta T \quad (53)
\]

\[
H_1 = 40 \frac{0.02 \times 0.02}{0.3} (100 - 0) = 5.33\text{Watts} \quad (54)
\]

Now for the Al bar

\[
H_2 = k \frac{A}{L} \Delta T \quad (55)
\]

\[
H_2 = 200 \frac{0.02 \times 0.02}{0.3} (100 - 0) = 26.66\text{Watts} \quad (56)
\]

So, for the total heat in this configuration

\[
H_{tot} = H_1 + H_2 = 32\text{Watts} \quad (57)
\]

(b) Calculate the total heat transfer rate, \( H \), for experiment B.

**ANSWER:** This is the harder one since we don’t know the temperature in the connection point between the two bars. We do, however, know that the heat flow through the bars must be the same:

\[
H = k \frac{A}{L} \Delta T \quad (58)
\]

\[
H = 40 \frac{0.02 \times 0.02}{0.3} (100 - T) \quad (59)
\]

\[
H = 0.053333 (100 - T) \quad (60)
\]

\[
H = 200 \frac{0.02 \times 0.02}{0.3} (T - 0) \quad (61)
\]

\[
H = 0.26666T \quad (62)
\]

\[
0.26666T = 5.3333 - 0.053333T \quad (63)
\]
\[ 0.32T = 5.3333 \quad (64) \]
\[ T = 16.666^\circ C \quad (65) \]
\[ H = 0.26666T = 4.44\text{Watts} \quad (66) \]

(c) Calculate the temperature at the place where the two bars connect in experiment B.

Already found during previous step.
8. 2 moles of Nitrogen, \(N_2\), gas are taken through the cycle shown in the figure below \((a \rightarrow b \rightarrow c)\). At point a, the temperature is 300 K and the pressure is 100,000 Pa. During the process \(a \rightarrow b\), the pressure of the system triples.

(a) Calculate and fill in the “state table” below:

**ANSWER:** Well, a few of the variables are simply given:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>V</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100000</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>b</td>
<td>300000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>100000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can use \(PV = nRT\) to get the Volume at a:

\[
V_a = \frac{nRT_a}{P_a} = 0.04989 \text{m}^3
\]  

(67)

This is also the volume at b.

Now we can use either \(PV = nRT\) or we can say that when the pressure triples while the volume is constant that the temp triples. This gives

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>V</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100000</td>
<td>0.04989</td>
<td>300</td>
</tr>
<tr>
<td>b</td>
<td>300000</td>
<td>0.04989</td>
<td>900</td>
</tr>
<tr>
<td>c</td>
<td>100000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OK, now is the only hard one. Points b and c are connected by an adiabatic curve. This means that:

\[
P_b V_b^\gamma = P_c V_c^\gamma
\]  

(68)
\[ \gamma = 7/5 = 1.4 (\text{diatomic gas}) \]  
\[ V_c^\gamma = \frac{P_b}{P_c} V_b^\gamma \]  
\[ V_c^\gamma = 3 \times V_b^\gamma \]  
\[ V_c = 3^{1/\gamma} \times V_b \]  
\[ V_c = 3^{0.714} \times V_b \]  
\[ V_c = 2.19 \times 0.04989 \]  
\[ V_c = 0.10935 m^3 \]  

This using PV=nT,

\[ T = \frac{PV}{nR} = \frac{100000(0.10935)}{2(8.315)} = 657.54 K \]  

This gives us the full table:

<table>
<thead>
<tr>
<th>P</th>
<th>V</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100000</td>
<td>0.04989</td>
</tr>
<tr>
<td>b</td>
<td>300000</td>
<td>0.04989</td>
</tr>
<tr>
<td>c</td>
<td>100000</td>
<td>0.10935</td>
</tr>
</tbody>
</table>

(b) Calculate and fill in the “process table” below:

**Answer:** The easiest thing to do as a starting point is to fill in the zeroes. There are two:

<table>
<thead>
<tr>
<th>W</th>
<th>Q</th>
<th>( \Delta U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a \rightarrow b</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b \rightarrow c</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c \rightarrow a</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The second easiest thing is to get the \( \Delta U \) values since the formula is the same for all processes:

\[ \Delta U = nC_V \Delta T \]  
\[ C_V = \frac{5}{2} R (\text{diatomic gas}) \]  
\[ a \rightarrow b : \Delta U = nC_V \Delta T \]  
\[ \Delta U = 2 \frac{5}{2} \times 8.315 (900 - 300) = 24945 J \]  
\[ b \rightarrow c : \Delta U = nC_V \Delta T \]  
\[ \Delta U = 2 \frac{5}{2} \times 8.315 (657.54 - 900) = -10080 J \]  
\[ c \rightarrow a : \Delta U = nC_V \Delta T \]  
\[ \Delta U = 2 \frac{5}{2} \times 8.315 (300 - 657.54) = -14865 J \]
Then the table becomes:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
<th>ΔU</th>
</tr>
</thead>
<tbody>
<tr>
<td>a → b</td>
<td>0</td>
<td>24945</td>
<td></td>
</tr>
<tr>
<td>b → c</td>
<td>0</td>
<td>-10080</td>
<td>-14865</td>
</tr>
<tr>
<td>c → a</td>
<td>0</td>
<td>-14865</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

OK, now we have 2/3 values for ab and bc, we can get the rest via

\[ ΔU = Q + W. \]

This yields:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
<th>ΔU</th>
</tr>
</thead>
<tbody>
<tr>
<td>a → b</td>
<td>0</td>
<td>24945</td>
<td>24945</td>
</tr>
<tr>
<td>b → c</td>
<td>0</td>
<td>-10080</td>
<td>-10080</td>
</tr>
<tr>
<td>c → a</td>
<td>0</td>
<td>-14865</td>
<td>-14865</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Next we can see that we need either the W or the Q for the ca process. We'll choose to calculate Q:

\[ Q = nC_P ΔT \] (86)
\[ C_P = \frac{7}{2} R(\text{diatomic gases}) \] (87)
\[ Q = n \frac{7}{2} RΔT \] (88)
\[ Q = \frac{7}{2} \times 8.315(300 - 657.54) = -20811J \] (89)
\[ ΔU = Q + W \] (90)
\[ W = ΔU - Q = -14865 - (-20811) = +5946J \] (91)

OK, then the table has:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
<th>ΔU</th>
</tr>
</thead>
<tbody>
<tr>
<td>a → b</td>
<td>0</td>
<td>24945</td>
<td></td>
</tr>
<tr>
<td>b → c</td>
<td>0</td>
<td>-10080</td>
<td>-14865</td>
</tr>
<tr>
<td>c → a</td>
<td>0</td>
<td>-20811</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Finally we can finish up the totals:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
<th>ΔU</th>
</tr>
</thead>
<tbody>
<tr>
<td>a → b</td>
<td>0</td>
<td>24945</td>
<td>24945</td>
</tr>
<tr>
<td>b → c</td>
<td>0</td>
<td>-10080</td>
<td>-10080</td>
</tr>
<tr>
<td>c → a</td>
<td>0</td>
<td>-20811</td>
<td>-14865</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0</td>
<td>4134</td>
<td>0</td>
</tr>
</tbody>
</table>

HOORAY!!! W=Q and ΔU = 0. HOORAY!!!

(c) Calculate the heat from the hot reservoir, \( Q_H \), and the heat to the cold reservoir, \( Q_C \).

**Answer:** \( Q_H \) is the sum of all positive heat flows. \( Q_C \) is the sum of all negative heat flows.

\[ Q_H = 24945J \] (92)
\[ Q_C = -20811J \] (93)

(d) Calculate the efficiency of your engine.
\textbf{Answer:} As always, efficiency is what you get over what you pay for. You get work from the engine and you pay for it with $Q_H$. 

$$
\epsilon = \frac{W}{Q_H} = \frac{-4134}{24945} = 0.1657 = 16.57\% \tag{94}
$$

(e) Calculate the efficiency of a Carnot engine operating between the same two extremes of temperature.

\textbf{Answer:} OK, finally we calculate the carn. The extremes of temperature are 300 and 900:

$$
\epsilon_{\text{Carnot}} = \frac{T_H - T_C}{T_H} = \frac{600}{900} = 0.6667 = 66.67\% \tag{95}
$$

\textbf{Note:} A problem of this same type is guaranteed to appear on the final exam.
9. Most people hang their plants from a string or a chain (and some of them talk to the plants). You decide instead to hang your plant from a spring \( k = 8 \frac{N}{m} \) so that it can have fun oscillating up and down. After you hang your plant, you start it bouncing with an amplitude of 12 cm. You see that it oscillates with a frequency of 2 Hz.

(a) What is the period of your plant? **ANSWER:**

\[ \tau = \frac{1}{f} = 0.5 \text{sec} \quad (96) \]

(b) What is \( \omega \)? **ANSWER:**

\[ \omega = 2\pi f = 4\pi = 12.57 \frac{rad}{sec} \quad (97) \]

(c) What is the maximum velocity of the plant? **ANSWER:**

\[ v_{max} = \omega A = 12.570.12 = 1.51 \frac{m}{s} \quad (98) \]

(d) What is the maximum acceleration? **ANSWER:**

\[ a_{max} = \omega^2 A = 12.57^20.12 = 18.96 \frac{m}{s^2} \quad (99) \]

(e) What is the velocity of the plant when its displacement from equilibrium is 4 cm? **ANSWER:**

\[
\left(\frac{x}{A}\right)^2 + \left(\frac{v}{\omega A}\right)^2 = 1 \\
\left(\frac{0.04}{0.12}\right)^2 + \left(\frac{v}{1.51}\right)^2 = 1 \\
\left(\frac{1}{3}\right)^2 + \left(\frac{v}{1.51}\right)^2 = 1 \\
\left(\frac{1}{9}\right) + \left(\frac{v}{1.51}\right)^2 = 1 \\
\left(\frac{v}{1.51}\right)^2 = \frac{8}{9} \\
\frac{v}{1.51} = \sqrt{\frac{8}{9}} \\
v = 1.51 \sqrt{\frac{8}{9}} = 1.42 \frac{m}{s} 
\]

15
(f) What is the acceleration of the plant when its velocity is $\frac{1}{2}v_{max}$? 

**ANSWER:**

$$\left(\frac{a}{\omega^2 A}\right)^2 + \left(\frac{v}{\omega A}\right)^2 = 1$$  
(107)

$$\left(\frac{a}{18.96}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$$  
(108)

$$\left(\frac{a}{18.96}\right)^2 + \frac{1}{4} = 1$$  
(109)

$$\left(\frac{a}{18.96}\right)^2 = \frac{3}{4}$$  
(110)

$$a = 18.96 \sqrt{\frac{3}{4}} = 16.42 \frac{m}{s^2}$$  
(112)

(g) What amplitude of the oscillation would produce a maximum acceleration of $9.8 \ \frac{m}{s^2}$?  

**ANSWER:**

$$a_{max} = \omega^2 A$$  
(113)

$$9.8 = 12.57^2 A$$  
(114)

$$A = 0.062 m$$  
(115)

(h) How far is the spring stretched when the plant is at equilibrium?  

**ANSWER:**

$$F = -kx$$  
(116)

$$mg = kx$$  
(117)

$$x = \frac{mg}{k} = \frac{g}{k/m} = \frac{g}{\omega^2} = 0.062 m$$  
(118)

**NOTE:** When your plant’s acceleration equals $9.8 \ \frac{m}{s^2}$ downward your plant feels weightless. However, you need not worry about vomit since the plant has no stomach.
10. Now that you have your medical degree, you decide to “hang a shingle” in front of your home and begin your practice. In Chapter 12, I would have asked you the following:

(a) What is the tension in the wire?
(b) What is the normal force from the wall?
(c) What is the upward force from the wall?

Now in Chapter 14 I will instead ask you different questions!! As shown, the length of the shingle is 20 cm. When a slight breeze blows, the shingle starts swinging with a small amplitude.

(a) What is the period of the motion of the shingle?

**Answer:** Well,...did you try the other parts or not :)

\[ \omega = \sqrt{\frac{mgd}{I}} \]  \hspace{1cm} (119)

\[ \omega = \sqrt{\frac{mgL}{3mL^2}} \]  \hspace{1cm} (120)

\[ \omega = \sqrt{\frac{gL}{5L^2}} \]  \hspace{1cm} (121)

\[ \omega = \sqrt{\frac{3g}{2L}} = \sqrt{3} \times 9.82 \times 0.2 = 8.57 \frac{rad}{s} \]  \hspace{1cm} (122)

\[ 8.57 \frac{rad}{s} = \frac{2\pi}{\tau} \]  \hspace{1cm} (123)

\[ \tau = \frac{2\pi}{8.57} = 0.733 \text{sec} \]  \hspace{1cm} (124)
(b) If the maximum angle of the shingle is $\theta_{\text{max}} = 0.01\text{rad}$, what is its maximum angular acceleration, $\alpha$? **ANSWER:**

$$\alpha_{\text{max}} = \omega_f^2 \theta_{\text{max}} = 8.57^2 \times 0.01 = 0.734 \frac{\text{rad}}{\text{s}^2}$$  \hspace{1cm} (125)

(c) What is the angular velocity, $\omega_v$, when the angle of the shingle is $\theta = 0.005\text{rad}$? **ANSWER:**

$$\left( \frac{\theta}{\theta_{\text{max}}} \right)^2 + \left( \frac{\omega_v}{\omega_f \theta_{\text{max}}} \right)^2 = 1$$  \hspace{1cm} (126)

$$\left( \frac{0.005}{0.01} \right)^2 + \left( \frac{\omega_v}{8.57 \times 0.01} \right)^2 = 1$$  \hspace{1cm} (127)

$$\left( \frac{1}{2} \right)^2 + \left( \frac{\omega_v}{0.0857} \right)^2 = 1$$  \hspace{1cm} (128)

$$\frac{1}{4} + \left( \frac{\omega_v}{0.0857} \right)^2 = 1$$  \hspace{1cm} (129)

$$\left( \frac{\omega_v}{0.0857} \right)^2 = \frac{3}{4}$$  \hspace{1cm} (130)

$$\frac{\omega_v}{0.0857} = \sqrt{\frac{3}{4}}$$  \hspace{1cm} (131)

$$\omega_v = 0.0857 \sqrt{\frac{3}{4}} = 0.0742 \frac{\text{rad}}{\text{s}}$$  \hspace{1cm} (132)

**NOTE:** Treat the shingle like a rod rotating about its end having a moment of inertia $I = \frac{1}{3}ML^3$.  

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11. An object oscillates according to the formula \( x(t) = 0.12 \cos(12t) \) where all numbers are in proper MKS units.

(a) What is the period of the motion? **ANSWER:**

\[
\omega = 12 \\
\frac{2\pi}{\tau} = 12 \\
\tau = \frac{2\pi}{12} = 0.5236s
\]

(b) What is the maximum velocity of the motion? **ANSWER:**

\[
v_{\text{max}} = \omega A = 12 \times 0.12 = 1.44 \frac{m}{s}
\]

(c) What is the velocity when \( x = 0.10 \) meters? **ANSWER:**

\[
\left( \frac{x}{A} \right)^2 + \left( \frac{v}{\frac{v}{1.44}} \right)^2 = 1 \\
\left( \frac{0.10}{0.12} \right)^2 + \left( \frac{v}{1.44} \right)^2 = 1 \\
0.69444 + \left( \frac{v}{1.44} \right)^2 = 1 \\
\left( \frac{v}{1.44} \right)^2 = 0.3055 \\
\frac{v}{1.44} = 0.553 \\
v = 1.44 \times 0.553 = 0.796 \frac{m}{s}
\]

(d) If this is the motion of a 0.100 kg mass on a spring, what is the spring constant? **ANSWER:**

\[
\omega = \sqrt{\frac{k}{m}} \\
\frac{k}{m} = \omega^2 \\
k = m\omega^2 = 0.1 \times 12^2 = 14.4 \frac{N}{m}
\]
12. A wave on a string is described by the equation \( y(x, t) = 0.25 \sin (3.14x + 450t) \), where all numbers are in proper MKS units.

(a) What is the frequency of the wave?

**ANSWER:**

\[
\omega = 450 = 2\pi f
\]

\[
f = \frac{450}{2\pi} = 71.62 \text{Hz}
\]

(b) What is the period of the wave?

(c) What is the wavelength of the wave?

**ANSWER:**

\[
k = 3.14 = \frac{2\pi}{\lambda}
\]

\[
\lambda = \frac{2\pi}{3.14} = 2.00 \text{m}
\]

(d) What is the velocity of the wave?

**ANSWER:**

\[
v = \lambda f = 2.00 \times 71.62 = 143 \frac{m}{s}
\]

(e) What is the amplitude of the wave?

**ANSWER:**

\[
A = 0.25 \text{meters}
\]

(f) Is the wave travelling in the positive or negative x direction? **ANSWER:** Since the sign between kx and \( \omega t \) is positive, the wave travels in the negative x direction.

(g) If the tension in the string is 100 N, what is the mass density, \( \mu \), of the string?

**ANSWER:**

\[
v = \sqrt{\frac{T}{\mu}}
\]

\[
v^2 = \frac{T}{\mu}
\]

\[
\mu = \frac{T}{v^2} = \frac{100}{143^2} = 0.00489 \frac{kg}{m}
\]
13. A flute should play an A (f=440 Hz) in its fundamental mode.

(a) Sketch the node pattern of the flute in its fundamental mode. 
**ANSWER:**

![Node Pattern](image)

(b) What is the length of the flute’s resonant cavity when sounding the A?
**ANSWER:**

\[
f_n = \frac{n v}{2L} \quad (155)
\]
\[
n = 1 \quad (156)
\]
\[
v = 345 \frac{m}{s} \quad (157)
\]
\[
440 = \frac{1}{2L} 345 \quad (158)
\]
\[
2L = \frac{345}{440} \quad (159)
\]
\[
L = \frac{345}{880} = 0.392m \quad (160)
\]

(c) To sound “middle C” (f=262 Hz) the flute player closes holes on the flute making it longer. How long to sound middle C?

**ANSWER:**

\[
f_n = \frac{n v}{2L} \quad (161)
\]
\[
n = 1 \quad (162)
\]
\[
v = 345 \frac{m}{s} \quad (163)
\]
\[
262 = \frac{1}{2L} 345 \quad (164)
\]
\[
2L = \frac{345}{262} \quad (165)
\]
\[
L = \frac{345}{524} = 0.658m \quad (166)
\]

(d) To sound a high A (f=880 Hz), the flute player cannot simply open more holes since the length will become too short? What do you do instead? Sketch the resulting node pattern.

**ANSWER:**
14. The “A” string on a guitar is the second lowest string (E is the lowest). This one ideally plays an A ($f=110$ Hz) two octaves below the A from the previous problem. The length of the string is 0.648 meters and the tension is 100 N.

(a) Sketch the node pattern when the string vibrates in its fundamental mode. **ANSWER:**

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad n = 1 \]  

\[ f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \]  

\[ f_1 \times 2L = \sqrt{\frac{T}{\mu}} \]  

\[ f_1 \times 2L = \sqrt{\frac{T}{\mu}} \]  

\[ (2fL)^2 = \sqrt{\frac{T}{\mu}} \]  

\[ \mu = \frac{T}{(2fL)^2} = 0.00492 \frac{kg}{m} \]  

(b) Find the mass per unit length, $\mu$, of the string. **ANSWER:**

(c) You wish to play a “C” ($f=131$ Hz) on this string. How do you accomplish this without changing the tension or $\mu$ of the string? Specify your answer as an exact numerical result.

**ANSWER:** To change the pitch you place a finger on the string and change its length. So, to give an exact answer, we determine the new length:

\[ f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \]  

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\[ L = \frac{1}{2f} \sqrt{\frac{T}{\mu}} \]  
(175) 

\[ L = 0.544m \]  
(176)
15. Two successive overtones of a closed organ pipe are 280 and 320 Hz.

(a) What is the fundamental frequency?

**Answer:** Since for a closed organ pipe, \( n=1,3,5,7... \) successive harmonics are separated by *twice* the fundamental frequency:

\[
2 \times f_1 = 320 - 280 \text{Hz} = 40 \text{Hz} \tag{177}
\]

\[
f_1 = 20 \text{Hz} \tag{178}
\]

\[
f_1 = \frac{1}{4L} v \tag{179}
\]

\[
20 = \frac{1}{4L} 345 \tag{180}
\]

\[
L = \frac{345}{80} = 4.31 \text{meters} \tag{181}
\]

(b) What is the length of the organ pipe?

**Answer:** Oops! I already answered that :P. BTW 20 Hz is the limit of the lowest frequency the average person can hear.
16. You are standing 10 meters away from a jet plane and the sound level of 140 dB is painful. Where should you stand to reduce the sound level to 120 dB?

**Answer:** You need to reduce the sound level by 20 dB. 20 dB means a factor of 100 as shown by:

\[-20\text{dB} = 10\log_{10}(\text{factor})\]  
\[-2 = \log_{10}(\text{factor})\]  
\[\text{factor} = 10^{-2} = \frac{1}{100}\]  

We know that intensity is inversely proportional to radius squared

\[I \propto \frac{1}{r^2}\]  
\[Ir^2 = \text{const}\]  
\[I_1r_1^2 = I_2r_2^2\]  
\[\frac{I_1}{I_2}r_1^2 = r_2^2\]  
\[r_2^2 = 100 \times 10^2 = 10000\]  
\[r_2 = 100\text{ meters}\]
17. Shown in the figure below is a submarine chasing a squid. Luckily for the squid he is moving faster. The submarine emits a sonar pulse with a frequency of 12,000 Hz. Using 1450 m/s as the speed of sound in water find all the following:

(a) Frequency of sound heard by the squid.

**ANSWER:** In this step, the source is the sub and the listener is the squid. In that case, the positive direction (from source to listener) is to the left. **BOTH** the squid and the sub therefore have positive velocities.

\[
f_{\text{squid}} = \frac{v_w - v_s}{v_w - v_s} f_{\text{sub}} \tag{191}
\]

\[
f_{\text{squid}} = \frac{1450 - 20}{1450 - 15} \times 12000 \tag{192}
\]

\[
f_{\text{squid}} = 11958 \text{ Hz} \tag{193}
\]

(b) Frequency of sound heard by the sub reflecting off the squid.

**ANSWER:** In this step, the source is the squid and the listener is the sub. In that case, the positive direction (from source to listener) is to the right. **BOTH** the squid and the sub therefore have negative velocities.

\[
f_{\text{squid}} = \frac{v_w - v_s}{v_w - v_s} f_{\text{sub}} \tag{194}
\]

\[
f_{\text{squid}} = \frac{1450 - 15}{1450 - 20} \times 11958 \tag{195}
\]

\[
f_{\text{squid}} = \frac{1465}{1470} \times 11958 \tag{196}
\]

\[
f_{\text{squid}} = 11917.5 \text{ Hz} \tag{197}
\]

(c) Beat frequency heard at the sub.
**Answer:** The sub hears the original 12000 Hz and also the final 11917.5 Hz:

\[
f_{\text{beat}} = f_1 - f_2 = 12000 - 11917.5 = 82.5 \text{ Hz}
\]  
(198)