Does $M_T$ Scaling Work at High $P_T$?

Jürgen Schaffner-Bielich

with Dima Kharzeev, Larry McLerran, and Raju Venugopalan

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Scaling relations

Initial momentum distribution of produced gluons (Color Glass Condensate):

\[ \frac{1}{\sigma} \frac{dN_g}{d\eta d^2p_t} = \frac{1}{\alpha_s(Q_s^2)} f_g \left( \frac{p_t^2}{Q_s^2} \right) \]

\(f_g\): universal, dimensionless function for produced gluons

\(\sigma\): transverse area of the two colliding nuclei

\(Q_s^2\): saturation scale for gluons, depends on energy, centrality, and atomic number

[Krasnitz and Venugopalan, PRL 86 (2001) 1717]
Assumed dynamical picture:

→ Initial gluon distribution characterized by saturated gluons
→ free streaming evolution producing additional partons
→ freeze-out to hadrons

scaling properties of initial state are preserved:

universal function $f_g \rightarrow f$ (hadrons)
saturation momentum $Q_s \rightarrow p_s$ (hadrons)

Note: $Q_s \neq p_s$ but $p_s$ has same energy, centrality
dependence as $Q_s$

Picture works for multiplicity and $\eta$ distributions at RHIC

[Kharzeev, Nardi, PLB507, 121 (2001); Kharzeev, Levin, nucl-th/0108006]

Question: can we describe the $p_t$ spectra (without hydrodynamical flow) in that picture?
Parameterize the transverse momentum distribution of hadrons as:

\[
\frac{1}{\sigma} \frac{dN_H}{dy d^2m_t} = \frac{1}{\alpha_s(p_s)} \kappa_h \cdot f \left( \frac{m_t}{p_s} \right)
\]

- \(f\): universal, dimensionless function for produced hadrons,
  depends on \(m_t\) (\(m_t\) scaling) and \(p_s\)

- \(\sigma\): transverse area of the two colliding nuclei

- \(p_s\): saturation scale for hadrons – has same centrality, energy, atomic number dependence as \(Q_s\)

- \(\kappa_h\): particle abundance (constant)

parameters \(\sigma\) and \(p_s\) to be determined from the empirical centrality dependence
Generalized scaling for identified hadrons?

Plot transverse mass spectra as function of $m_t$ (not $p_t$, not $m_t - m_0$):

curves are close to each other without any adjustments in absolute normalization!

generalized $m_t$-scaling: $m_t$ spectra follows more a power law than an exponential form!
curves for different hadrons are shifted $\rightarrow$ same local slope at given $m_t$?
Answer: Change absolute normalization

for protons: $\kappa_p = 2$, for kaons: $\kappa_K = 1/2$

curves are now on top of each other, slopes are identical for the same given $m_t$

constant $\kappa_h$ is indeed of order one, kaons are suppressed, protons are enhanced

seems to work also for high $m_t$ where particle ratios are given by the constants $\kappa_h$
Check the details: linear plot

divide by power law fit:

$$\frac{1}{(1 + m_t/p_s)^n}, \ p_s = 2.71 \ \text{GeV}, \ n = 16.3$$

even with systematic errors from fit deviations are of the order of 30% only!
Local slope parameter:

Assume the scaling function to be a power law:

$$ f \left( \frac{m_t}{p_s} \right) \sim \left( 1 + \frac{m_t}{p_s} \right)^{-n} $$

and define the local slope as

$$ - \frac{1}{T_{\text{slope}}} = \frac{d}{dm_t} \ln (f(m_t/p_s)) $$

$$ T_{\text{slope}} = \frac{p_s}{n} + \frac{1}{n} m_t $$

looks like hadronic flow formula, i.e. a constant term plus a term linear in $m_t$!

term proportional to $m_t$ originates from the non-exponential behavior of the power-law

(fit to $m_t$ spectra with $n = 16.3$, $p_s = 2.71$ GeV gives:

$T_\pi = 175$ MeV, $T_K = 196$ MeV, $T_p = 224$ MeV,

$T_\Lambda = 235$ MeV, $T_{\Xi} = 247$ MeV)
Scaling with centrality

generalized $m_t$ scaling for all centralities?
check for $\pi^-$ and $\bar{p}$:

$\Rightarrow \pi^-$ and $\bar{p}$ data form a continuous curve for each centrality bin
Particle spectra at RHIC

check universality of scaling function \( f \) by rescaling of \( dN/dy dp_t^2 \) and \( m_t \), so that data of different centralities are on top of each other:

\[
\frac{dN_h}{dy d^2m_t} = \frac{\sigma}{\alpha_s(p_s)} \kappa_h \cdot f \left( \frac{m_t}{p_s} \right)
\]
Rescaled $m_t$ distributions

rescale different centrality bins as

$$\frac{1}{\sigma} \frac{dN_h}{dy d^2 m_t} \rightarrow \frac{1}{\lambda} \frac{1}{\sigma} \frac{dN_h}{dy d^2 m_t}$$

and

$$m_t \rightarrow \frac{m_t}{\lambda}$$

$\Rightarrow$ one universal function $f$ describing all centrality bins!
Check the details for centrality dependence:

divide by power law fit:

\[
1/ (1 + m_t/p_s)^n, \quad p_s = 1.65 \text{ GeV}, \quad n = 11.8
\]

works within 30% up to \( m_t = 2 \text{ GeV} \)!
Scaling of the transverse area

scaling factor $\sigma/\alpha_s(p_s)$ relative to most central bin ($N_{\text{part}} = 347$):

![Graph showing scaling factor $\sigma/\alpha_s$ vs. $N_{\text{part}}$]

expect to scale like

$$N_{\text{part}}^{2/3}/\alpha_s(p_s) \sim N_{\text{part}}^{2/3} \ln \left( \frac{p_s^2}{\Lambda_{\text{QCD}}^2} \right)$$

where

$$p_s^2/p_{s,c}^2 \sim c + c' \cdot N_{\text{part}}^{1/3}$$

as taken from centrality dependence of $p_s$
Scaling of the transverse momentum parameters $p_s$ normalized to most central bin:

![Graph showing scaling factor $p_s$ vs $N_{\text{part}}$]

expect to scale like $Q_s \sim N_{\text{part}}^{1/6}$ but

$$p_s^2/p_{s,c}^2 = c + c' \cdot N_{\text{part}}^{1/3} = 0.61 + 0.39 \left( N_{\text{part}}/347 \right)^{1/3}$$

compatible with transverse energy per charged particle

(PhENIX coll., PRL 87, 0523201 (2001))

constant $c$ stands for the finite transverse momenta for pp collisions ($\langle p_t \rangle \sim p_s$),

compatible with $\langle p_t \rangle$ reported by UA1 and STAR

(392/508 = .77)
dN/dp_{t} (a.u.)

p_{t} (GeV)

h^{+}+h^{-}

0-5%
5-15%
15-30%
30-60%
60-80%
80-92%

PHENIX
Check the details for charged hadron spectra and $m_t$ scaling:

divide charged hadron data by $m_t$ power law:

$$\frac{dN}{d\eta} \sim \frac{\sigma}{\alpha_s} \left[ \kappa_{\pi} f \left( \frac{m_{t,\pi}}{p_s} \right) + \kappa_K f \left( \frac{m_{t,K}}{p_s} \right) + \kappa_p f \left( \frac{m_{t,p}}{p_s} \right) \right]$$
Check scaling for charged hadron spectra:

divide charged hadron data by $p_t$ power law:

$$\frac{1}{(1 + p_t/p_s)^n}, \quad p_s = 3.24 \text{ GeV}, \quad n = 16.1$$
$p_t$ (GeV)

Central/Peripheral bin

$\times h^+ + h^-$

$\ast \pi^0$

scaling (h)

scaling ($\pi$)

PHENIX
Consequences for centrality dependence:

\[
\frac{1}{\sigma} \cdot \frac{dN}{dy} \sim \int_{m_{\text{vac}}}^{\infty} f \left( \frac{m_t}{p_{\text{scaling}}} \right) dm_t^2
\]

\[
\frac{1}{\sigma} \cdot \frac{dN}{dy} \sim p_{\text{scaling}}^2 \cdot F(m_{\text{vac}}/p_{\text{scaling}})
\]

- particle numbers increase as:

\[
\frac{1}{N_{\text{part}}} \frac{dN}{dy} \sim p_{\text{scaling}}^2 \sim <p_t>^2
\]

- particle ratios increase for more central collisions as \( p_{\text{scaling}} \) increases like:

\[
\frac{K}{\pi} < \frac{\bar{p}}{\pi} < \frac{\Lambda}{\pi} < \ldots
\]

(for the power law parameters we find increases from \( pp \) to \( \text{AuAu} \) of \( K/\pi : \bar{p}/\pi : \Lambda/\pi = 1.5 : 2.2 : 2.7 \))