3rd RHIC Update

QCD quenching

/ + Pout - Special /

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1. Broadening and Induced Radiation
2. Landau-Pomeranchuk-Migdal Radiation Suppression in QED and QCD
3. From medium-induced radiation to Quark
4. Infrared (in)Sensitivity
5. Heavy Quarks as a probe
6. Why all the Fuss?

7. Special: "intrinsic" headache
Common Quantum Mechanics is
\[1\] Gluons with large formation times are emitted coherently (<< than in BH)

LPM QCD physics on the back of env.

\[ t_{\text{form.}} \sim \frac{\omega}{q} \quad \text{AND} \quad k_T^2 \sim \frac{m^2 \cdot \frac{L}{\lambda}}{\gamma} = \frac{m^2 |Q|^2 t_{\text{form}}}{\gamma} \]

\[ t_{\text{form}} \sim \sqrt{\frac{\omega}{q}} \quad \text{AND} \quad k_T^2 \sim \sqrt{\omega q} \]

\[ \frac{w \, dN}{dw \, dz} = \left( \frac{x_s C_F}{\gamma} \right)^{BH} \sqrt{\frac{\mu_B^2 \lambda}{w}} \]

\[ \frac{dN}{dN_{BH}} \]

\[ \frac{w}{w_{BH}} \]
The gluon, during its \textit{formation time}

\[ t_{\text{form}} \simeq \frac{\omega}{k^2_\perp}, \]

accumulates a \textit{typical} transverse momentum

\[ k^2_\perp \simeq \mu^2 \frac{t_{\text{form}}}{\lambda}, \]

with $\lambda$ the mean free path and $\mu^2$ the characteristic momentum transfer squared in a single scattering.

\[ N_{\text{coh}} = \frac{t_{\text{form}}}{\lambda} = \sqrt{\frac{\omega}{\mu^2 \lambda}} \]

describes the number of scattering centres which participate, \textit{coherently}, in the gluon emission with a given energy $\omega$.

\[ \langle k^2_\perp \rangle \simeq \sqrt{\frac{\mu^2}{\lambda}} \cdot \omega, \quad \langle \Theta^2 \rangle \simeq \sqrt{\frac{\mu^2}{\lambda}} \cdot \omega^{-3} \]
Suppression of the Bethe-Heitler radiation spectrum

\[ \ln \left( \frac{\lambda_{\text{int}} I}{d\omega d\Omega} \right) \]

QED \quad \sqrt{\omega} \quad \text{B.H.}

\[ QCD \quad \frac{1}{\sqrt{\omega}} \]

QED radiation \quad \text{VANISHES in the } E \to \infty \quad \text{at } \omega \to \infty

QCD \quad \text{STAYS}

\[ j \quad j' \quad \theta_s \]

\[ j'' \quad j'\prime \quad \theta_s \text{ may be zero, still } \delta j \neq 0 \quad [\text{COLOR}!] \]
For sufficiently large gluon energies, \( \omega > \mu^2 \lambda \) (the coherent length exceeds the mean free path, \( N_{coh} > 1 \)), the standard Bethe-Heitler energy spectrum per unit length describing independent emission of gluons at each centre gets suppressed:

\[
\frac{dW}{d\omega d\Lambda} = \left( \frac{dW}{d\omega d\Lambda} \right)^{BH} \cdot \frac{1}{N_{coh}} = \frac{\alpha_s C_R}{\pi \omega \lambda} \cdot \frac{\sqrt{\mu^2 \lambda}}{\omega} = \frac{\alpha_s C_R}{\pi \omega} \sqrt{\frac{\hat{q}}{\omega}}.
\]

The gluon transport coefficient:

\[
\hat{q} \equiv \frac{\mu^2}{\lambda} = \rho \int \frac{d\sigma}{dq^2} q^2 dq^2,
\]

is proportional to the density \( \rho \) of the scattering centres in the medium and describes the typical momentum transfer in the gluon scattering off these centres.

The transport coefficient for cold nuclear matter (BDMPS):

\[
\hat{q} \simeq \frac{4\pi^2 \alpha_s N_c}{N_c^2 - 1} \rho \left[ xG(x, Q^2) \right], \quad Q^2 \sim \hat{q} L,
\]

with \( \rho \simeq 0.16 \text{ fm}^{-3} \) the average nuclear density and \( [xG(x)] \) the gluon density in a nucleon.
Choosing $L \sim 5$ fm results in $Q \sim 0.5$ GeV. Taking at this scale $\alpha_s \sim 0.5$ and $[xG(x)] \sim 1$ (at $x < 0.1$), yields

$$\hat{q}_{\text{cold}} \sim 0.01 \text{ GeV}^3 \sim 8 \rho.$$  

Is in agreement with the result of the analysis of the gluon $p_{\perp}$ broadening from the experimental data on $J/\psi$ transverse momentum distributions,

$$\hat{q} = (9.4 \pm 0.7) \rho.$$  

An estimate for a hot medium based on perturbative treatment of gluon scattering in quark–gluon plasma with $T \sim 250$ MeV resulted in the value of the gluon transport coefficient of about factor twenty larger:

$$\hat{q}_{\text{hot}} \sim 0.2 \text{ GeV}^3 \sim 20 \hat{q}_{\text{cold}}.$$
Energy Spectrum:

\[
\frac{dI(\omega)}{d\omega} = \frac{\alpha}{\omega} \ln \left| \cos \sqrt{\frac{i\omega_c}{\omega}} \right| = \frac{\alpha}{2\omega} \ln \left[ \cosh^2 \sqrt{\frac{\omega_c}{2\omega}} - \sin^2 \sqrt{\frac{\omega_c}{2\omega}} \right];
\]

\[\alpha \equiv \frac{2\alpha_s C_R}{\pi}.\]

The characteristic gluon energy parameter

\[\omega_c = \frac{\hat{q}}{2} L^2\]

The distribution peaks at small gluon energies \((\omega < \omega_c)\),

\[
\omega \frac{dI(\omega)}{d\omega} = \alpha \left\{ \sqrt{\frac{\omega_c}{2\omega}} - \ln 2 \right\} \left[ 1 + \mathcal{O} \left( \exp \left\{ -\sqrt{\frac{2\omega_c}{\omega}} \right\} \right) \right],
\]

while for energies above the characteristic scale \(\omega_c\) it is small and falling fast with \(\omega\):

\[
\omega \frac{dI(\omega)}{d\omega} \sim \frac{\alpha}{12} \left( \frac{\omega_c}{\omega} \right)^2, \quad \omega > \omega_c.
\]
Distribution in accumulated energy loss

\[ D(\epsilon) = \int_C \frac{d\nu}{2\pi i} \tilde{D}(\nu) e^{\nu \epsilon}, \]

In the Mellin space, summing up multi-gluon radiation gives

\[ \tilde{D}(\nu) = \exp \left[ - \int_0^\infty d\omega \frac{dI(\omega)}{d\omega} \left( 1 - e^{-\nu \omega} \right) \right]. \]

Can be expressed via integrated gluon multiplicity \( N(\omega) \):

\[ \tilde{D}(\nu) = \exp \left[ - \int_0^\infty dz \frac{e^{-z}}{N \left( \frac{z}{\nu} \right)} \right]. \]

Finally, using the fact that the vacuum cross section \( d\sigma^{\text{vacuum}}(p_\perp) \propto p_\perp^{-n} \) with \( n \approx 10 - 12 \) (RHIC),

\[ \frac{d\sigma^{\text{vacuum}}(p_\perp + \epsilon)}{dp_\perp^2} \sim \frac{d\sigma^{\text{vacuum}}(p_\perp)}{dp_\perp^2} \cdot \exp \left\{ -\epsilon \frac{n}{p_\perp} \right\}, \]

the Quenching Factor emerges,

\[ Q(p_\perp) = \left[ \frac{d\sigma^{\text{vac}}(p_\perp)}{dp_\perp^2} \right]^{-1} \int d\epsilon D(\epsilon) \frac{d\sigma^{\text{vac}}(p_\perp + \epsilon)}{dp_\perp^2} \]

\[ \sim \tilde{D} \left( \nu = \frac{n}{p_\perp} \right). \]
Probability of not losing anything violently suppressed:

$$\varepsilon D(\varepsilon) \sim \alpha \sqrt{\frac{\omega_e}{2\varepsilon}} \ e^{-\frac{\pi a^2 \omega_e}{\varepsilon}}$$

1st order, \(1^{st} \omega_e < \varepsilon < \omega_c\)

An interplay between falling X-section (wanting \(\varepsilon \) small) and the Form Factor (\(D(\varepsilon)\) pushing out away from \(\varepsilon = 0\))
\[ \frac{d\sigma^{\text{medium}}(p_\perp)}{dp_\perp^2} = \frac{d\sigma^{\text{vacuum}}(p_\perp + S)}{dp_\perp^2}. \]

The *shift* parameter \(S\) is usually taken \(S = \text{const} \cdot L\), or equal to the *mean* medium induced energy loss

\[ S = \Delta E \equiv \int d\epsilon \epsilon D(\epsilon) \propto \alpha_s L^2. \]

The former ansatz has no theoretical justification while the latter misses one essential point, namely that the vacuum distribution is a *sharply falling function of* \(p_\perp\). A strong bias suppresses real gluon radiation, and the *typical* energy carried by accompanying gluons turns out to be much smaller than the *mean*:

\[ S(p_\perp) \approx \sqrt{\frac{2\pi \alpha^2 \omega_c p_\perp}{n}}. \]

**Normalized shift**
Quenching factors for cold nuclear matter. The curves (from bottom to top) correspond to the gluon energy cuts 0, 100, 300 and 500 MeV.
“Infrared” dependence of the quenching factor for hot medium. The curves (from bottom to top) correspond to the gluon energy cuts 0, 100, 300 and 500 MeV.
The curves (from bottom to top) correspond to the gluon energy cuts 0, 100, 300 and 500 MeV.
\[ \ln Q(p_\perp) \propto -\alpha \int_0^\infty \frac{d\omega}{\omega} \left[ 1 - \exp \left( \frac{n\omega}{p_\perp} \right) \right] \times \sqrt{\frac{qL^2}{\omega}} \]

Formally convergent, … BUT:

\[ \sqrt{\frac{qL^2}{\omega}} \sim \sqrt{\frac{\omega_{BH}}{\omega}} \cdot \frac{L}{\lambda} \quad \omega_{BH} \sim \mu^2 \lambda \sim q\lambda^2. \]

The integration region \( \omega \lesssim \omega_{BH} \) contributes

\[ \delta [\ln Q(p_\perp)] \sim \alpha \frac{n\omega_{BH}L}{p_\perp \lambda} \sim \alpha \frac{nL\mu^2}{p_\perp} \sim \alpha \frac{nT^2}{p_\perp} \cdot L \]

Double-enhanced “1/Q power correction”.

HUGE!

Extreme sensitivity to the spectral properties of the medium!

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naïve probe:

Cut off gluon energies below some \( \omega_{\text{min}} \)
Dead Cone

\[ dP = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{k_1^2 dk_1^2}{(k_1^2 + \omega^2 \theta_0^2)^2}, \quad \theta_0 = \frac{M}{E}, \]

with \( \alpha_s \) evaluated at the scale = denominator.

The ratio of the gluon spectra

\[ dP_{HQ} = dP_0 \cdot \left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \]

Suppression of small-angle radiation has a number of interesting implications, such as

\[ \times \] PT calculability of (and NP \( \Lambda/M \) corrections to) heavy quark fragmentation functions (D-r, Khoze, Troyan, Nason, Webber)

\[ \times \] multiplicity and

\[ \times \] energy spectra

of light particles accompanying hard production of a heavy quark (DKT)
the Dead-Cone suppression:
(D-r & Kharzeev)

\[ I(\omega) = \omega \frac{dW}{d\omega} = \alpha_s \frac{C_F}{\pi} \sqrt{\frac{\omega_1}{\omega}} \frac{1}{(1 + (\ell \omega)^{3/2})^2}, \]

where

\[ \ell \equiv \hat{q}^{-1/3} \left( \frac{M}{E} \right)^{4/3}. \]

The suppression factor \((x = \omega/p_\perp)\):

![Graph showing the ratio of gluon emission spectra off charm and light quarks for quark momenta \(p_\perp = 10 \text{ GeV}\) (solid line) and \(p_\perp = 100 \text{ GeV}\) (dashed); \(x = \omega/p_\perp\).]
Comparison of energy distributions $\sqrt{x}I(x)$ of gluons radiated off charm (solid line) and light (dashed line) quarks in hot matter with $\hat{q} = 0.2 \text{ GeV}^3$, $p_\perp = 10 \text{ GeV}$, $L = 5 \text{ fm}$.
Charm/light quarks in cold nuclear matter
\( \hat{q} = 0.01 \text{ GeV}^3, L = 5 \text{ fm}. \)
The ratio of quenching factors for charm and light quarks in hot matter with $\hat{q} = 0.2 \text{ GeV}^3$
Solid lines correspond to unrestricted gluon radiation, while the dashed lines are based on the calculation with the cut on gluon energies $\omega > 0.5 \text{ GeV}$. 

$L = 5 \text{ fm}$

$L = 2 \text{ fm}$
Conclusion:

Quenching of single inclusive hadron distributions though, FORMALLY, a CALCULABLE Collinear- and- Infrared- Stable (fin quantity, is EXTREMELY sensitive to actual spectral properties of the Final- State Medium (effective gluon "mass")?

\[ \int_0^\infty \frac{d\omega}{\sqrt{\omega}} = \text{const} < \infty \quad \text{BUT!} \]

Can be looked upon as an \( \Omega \)-enhanced "power correction" — confinement-sensitive

\[ p_1 \sim 5 \text{ GeV} \]

\[ p_1 \sim 20 \text{ GeV} \]

\[ 10^4 \% \]

\[ \frac{1}{100 \%} \]
Specials

"intrinsic" transverse momenta
Quantum Mechanics (WYSIWYG; S. Ellis)
and "Landau warning":

"Don't think .... ... fools"

Want to talk point-like partons?
— DON'T measure final-state structure

Historical example

Drell-Yan:  TOTAL X-section \( \frac{d\sigma}{dM} \)

vs

DIFFERENTIAL \( \frac{d\sigma}{dm^2 dp_T^2} \)

<table>
<thead>
<tr>
<th>( \frac{d\sigma}{dp_T^2} )</th>
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\( \frac{\alpha}{p_T^2} \)

\( \frac{9}{q} \)

Parton model

Fixed order QCD

ALL-order QCD

\( \frac{\alpha}{\mu} \)

\( \frac{\alpha}{p_T} \)

-- intrinsic ?!
Form Factor price to pay for
VETOING accompanying radiation

\[- \ln F \simeq C \int \frac{d\omega}{\omega} \int \frac{d^2 k_1}{2\pi k_1^2} \cdot \frac{\alpha_s (k_1^2)}{5} \]

Plateau in Drell-Yan : \( \frac{1}{p_1^2} \rightarrow \text{const} \)

\[ Q_0 = Q \frac{1}{1 + \frac{\beta}{2e}} \wedge \frac{1}{1 + \frac{2\beta}{\beta^2}} \]

\[ Q_0 = Q \sqrt{\frac{1}{1 + \frac{\beta}{2e}} \wedge \frac{1}{1 + \frac{2\beta}{\beta^2}}} \]

\[ Q^2 = M^2 \]

Plateau in 2-particle back-to-back correlation : \( \frac{L_{\text{out}}}{P_{\text{out}}} \rightarrow \text{const} \)

\[ Q_0 = Q \frac{1}{1 + \frac{\beta}{2e}} \wedge \frac{1}{1 + \frac{2\beta}{\beta^2}} \]

\[ \beta = \left( \frac{1}{3} n_c - \frac{2}{3} n_f \right) = 9 \]

\[ C = \sum C_i \]

Sum of Color Charges
DY: $c = \frac{4}{3} + \frac{4}{3}$  $q_0 \sim M^0.37$

$\frac{d\sigma}{dp_T^2}(x)$: $c = 3 + 3$  $q_0 \sim M^0.57$

$\frac{d\sigma}{dp_{out}}(\phi\phi)$: $c = \frac{4}{3} + \frac{4}{3}$  $q_0 \sim p_T^{0.50}$

$\frac{d\sigma}{dp_{out}}(h, h_2)$: $c = 4 \cdot \frac{4}{3}$  $q_0 \sim p_T^{0.7}$

$c = 4.3$  $q_0 \sim p_T^{0.8}$