\[4.1: a) \hat{\gamma} = \hat{x} + \hat{y} f + \hat{z} \mathbf{k}. \quad \hat{p} = P_x \hat{x} + P_y \hat{y} + P_z \hat{z}.\]
\[P_x = \frac{\partial}{\partial x}, \quad P_y = \frac{\partial}{\partial y}, \quad P_z = \frac{\partial}{\partial z}.\]
\[\{x, P_x\} f(x, y, z) = x \frac{\partial f}{\partial x} - \frac{\partial}{\partial x} f(x, y, z) = \frac{1}{2} \{x, \frac{\partial f}{\partial x}\} = (\nabla f).\]
\[\{x, y\} f(x, y, z) = xy f - yx f = 0\]
\[\{P_x, P_y\} f(x, y, z) = \frac{\partial^2 f}{\partial x \partial y} f - \frac{\partial^2 f}{\partial x \partial y} f = 0\]
In general, \[\{x, P_y\} = -\{P_x, y\} = \nabla \delta g, \quad \{x, P_y\} = \{P_x, P_y\} = 0\]

b) \[\frac{\partial}{\partial x} \hat{\gamma} = \frac{\partial}{\partial x} [\{L, \hat{\gamma}\} + \{P_y, \hat{\gamma}\}].\]
Let \(\hat{\gamma} = \hat{\gamma}\)
\[\frac{\partial}{\partial x} \hat{\gamma} = \frac{\partial}{\partial x} [L \{P_y, \hat{\gamma}\}] = \frac{\partial}{\partial x} [L \{P_y, \hat{\gamma}\}].\]
\[\{L, \hat{\gamma}\} = \{P_y, \hat{\gamma}\} = \frac{\nabla}{\nabla} (L \{P_y, \hat{\gamma}\}) = -\frac{\nabla}{\nabla} \hat{\gamma}.\]
Let \(\hat{\gamma} = \hat{\gamma}\)
\[\frac{\partial}{\partial x} \hat{\gamma} = \frac{\partial}{\partial x} [L \{P_y, \hat{\gamma}\}] = \frac{\partial}{\partial x} [L \{P_y, \hat{\gamma}\}].\]
Since \[\{L, \hat{\gamma}\} = (\nabla \nabla \hat{\gamma}), \quad \forall \hat{\gamma}, \quad \text{in three dimension we have},\]
\[\{\hat{\gamma}, \hat{\gamma}\} = (\nabla \nabla \hat{\gamma}).\]
\[\frac{\partial}{\partial x} \hat{\gamma} = \frac{\partial}{\partial x} [L \{P_y, \hat{\gamma}\}] = -\{P_y, \hat{\gamma}\}.\]

\[4.8: \text{AR} \delta_1 (k_r V) = A \hat{\delta}_1 (\frac{\delta_3 x}{\delta_3 k_r} - \frac{\cos k_r x}{k_r} ).\]
\[4.37: \text{with } V(r) = 0, \quad \text{and } l = 1 \text{ becomes:}\]
\[-\frac{\delta^2}{\delta y^2} U + \frac{\delta^2}{\delta y^2} U = EU.\]
\[\text{where } E = \frac{\delta^2}{\delta y^2} U.\]

Let \(x = k_r V, \) it turns to prove.
\[\delta_3 U (\cos x - \cos x) - \frac{3}{2} (\sin x - \cos x) + \sin x - \cos x = 0. \quad (**).\]
\( (k \lambda) \) is valid.

b) \( \frac{\sin x}{x} - \frac{\cos x}{x} = 0 \Rightarrow \pi x = x \)

plot the figures of \( f_1(x) = x \) and \( f_2(x) = \pi x \), their interest are the roots we're looking for.

\( x \to 0, \pi x \to 0 \). \( x = \left(n + \frac{1}{2}\right) \pi, (N \gg 1) \).

\[ E_{n1} = \frac{m_1 \omega^2}{2m_2} \left(n + \frac{1}{2}\right)^2 \] formula [4.50].