Parton Structure of the Proton

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March 30, 2015
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Quarks as constituents

- “A Schematic Model of Baryons and Mesons” (Gell-Mann, 1964)

How can structure be described?
How can structure be probed?
Probing structure with scattering

Elastic scattering

Pointlike target

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \cdot \left[ 1 + 2\tau \tan^2 \frac{\theta}{2} \right]
\]

Extended target

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \cdot \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2} \right]
\]

Form factors $G_E, G_M$ yield information about the proton as a whole
Deep inelastic scattering (DIS)

Relevant kinematic variables:
\[
\begin{align*}
\nu &= E - E' \\
y &= \frac{q \cdot P}{k \cdot P} \\
Q^2 &= -q^2 \gg M^2 \\
W^2 &= (P + q)^2 \\
x &= \frac{Q^2}{2P \cdot q} \\
s &= (k + P)^2
\end{align*}
\]

De Broglie wavelength \( \lambda = 1/p \)
\[ \to Q \text{ determines resolution} \]
Neutral current cross section

For incident particle/antiparticle:

\[
\frac{d^2 \sigma^{NC}}{dx dy} = \frac{2\pi\alpha^2}{xyQ^2} \left[ (1 + (1 - y)^2) F_2^{NC}(x, Q^2) \mp (1 - (1 - y)^2)x F_3^{NC}(x, Q^2) \right]
\]

where the structure function \( F_1^{NC}(x, Q^2) \) has been eliminated by using the Callan-Gross relation, \( 2x F_1^{NC}(x, Q^2) = F_2^{NC}(x, Q^2) \)

\[
F_2^{NC} = F_2^\gamma - \eta \gamma Z (g_V^e \pm \lambda g_A^e) F_2^{\gamma Z} + \eta Z (g_V^{e2} + g_A^{e2} \pm 2\lambda g_V^e g_A^e) F_2^Z
\]

\[
F_3^{NC} = -\eta \gamma Z (g_A^e \pm \lambda g_V^e) F_3^{\gamma Z} + \eta Z [2g_V^e g_A^e \pm \lambda (g_V^{e2} + g_A^{e2})] F_3^Z
\]
Charged current cross section

\[
\frac{d^2 \sigma^{CC}}{dx dy} = \frac{\pi \alpha^2}{xyQ^2} (1 \pm \lambda)^2 \left( \frac{G_F M_W^2}{4\pi \alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2 \\
\quad \times \left[ (1 + (1 - y)^2) F_2^{CC}(x, Q^2) \mp (1 - (1 - y)^2) x F_3^{CC}(x, Q^2) \right]
\]

\( W \) boson only carrier of the charged current \( \rightarrow F_i^{CC}(x, Q^2) = F_i^{W^\pm}(x, Q^2) \)
Bjorken scaling and $x$

Choose frame:

- Very high proton momentum ($P \to \infty$)

Take limit:

- $Q^2 \to \infty$
- $\nu \to \infty$
- $x = \frac{Q^2}{2P \cdot q}$ fixed

Consequences:

- $F(x, Q^2) \to F(x)$
- Bjorken $x$ is fractional momentum of interacting quark
Scaling observation

First measurement of $F_2^p(x)$ by MIT-SLAC (1968)
$e^-$ on hydrogen

Lack of $Q^2$ dependence suggests point like constituents
Quark-parton model

Structure functions are linear combinations of PDFs

\[ F_2^\gamma(x) = x \sum_f e_f^2 [q_f(x) + \bar{q}_f(x)] \]

\[ F_2^{\gamma Z} = x \sum_f 2e_f g_V^f [q_f(x) + \bar{q}_f(x)] \]

\[ F_2^Z = x \sum_f g_V^f q_f(x) + \bar{q}_f(x) \]

PDFs are universal $\rightarrow$ extract from measurements of structure functions!
Parton distribution functions (PDFs)

\[ q_f(x) = \text{probability of finding parton of flavor } f \text{ with fractional momentum } x \]

\[ Q^2 = 1.69 \text{ GeV}^2 \]

\[ q_f^{val} = q_f(x) - \bar{q}_f(x) \]

\[ p_f = \int_0^1 dx \ x [q_f(x) + \bar{q}_f(x)] \]
Summary of structure functions

\[ F_2^\gamma(x) = x \sum_f e_f^2 [q_f(x) + \bar{q}_f(x)] \]
\[ F_2^{\gamma Z}(x) = x \sum_f 2e_f g_V^f [q_f(x) + \bar{q}_f(x)] \]
\[ F_2^Z(x) = x \sum_f g_V^f [q(x) + \bar{q}(x)] \]

\[ F_2^{W^+}(x) = 2x \left[ \sum_{f_d} q_f(x) + \sum_{f_u} \bar{q}_f(x) \right] \]
\[ F_3^{W^+}(x) = 2 \left[ \sum_{f_d} q_f(x) - \sum_{f_u} \bar{q}_f(x) \right] \]

\[ F_2^{W^-}(x) = 2x \left[ \sum_{f_u} q_f(x) + \sum_{f_d} \bar{q}_f(x) \right] \]
\[ F_3^{W^-}(x) = 2 \left[ \sum_{f_u} q_f(x) - \sum_{f_d} \bar{q}_f(x) \right] \]
DIS experiments

HERA \((e^- p \text{ collider}) \rightarrow F_{2}^{NC}\)
- H1
- ZEUS

CERN \((\mu^+ \text{ on liquid hydrogen}) \rightarrow F_{2}^{NC}\)
- BCDMS
- EMC
- NMC

Fermilab
- E665 \((\mu^+ \text{ on liquid hydrogen}) \rightarrow F_{2}^{NC}\)
- NuTeV \((\nu_\mu, \bar{\nu}_\mu \text{ on steel}) \rightarrow F_{2}^{CC}\)
Another look at scaling

- $F_2(x) \neq F_2(x, Q^2)$
- *Scaling violation!*

In addition,

$$\sum_f \int_0^1 dx \ x [q_f(x) + \bar{q}_f(x)] \approx 0.5$$
NLO processes

$O(\alpha_s)$ processes

Gluon emission

Gluon scattering
Perturbative corrections

DGLAP equations

Including gluon emission and the gluon distribution $g(x, Q^2)$ leads to coupled differential equations:

$$\frac{d q(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ q(y, Q^2) P_{qq} \left( \frac{x}{y} \right) + g(y, Q^2) P_{qg} \left( \frac{x}{y} \right) \right]$$

$$\frac{d g(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ \sum_f [q_f(y, Q^2) + q_f(y, Q^2)] P_{gq} \left( \frac{x}{y} \right) + g(y, Q^2) P_{gg} \left( \frac{x}{y} \right) \right]$$

Splitting functions $P_{f,i}(z)$

- Probability of particle i in initial state, particle j (with momentum fraction $z$) in final state
- Calculated in QCD
Global fit

- Develop program to numerically solve DGLAP equations
- Choose experimental data sets to include
- Select parameterization for PDFs at input scale $Q_o^2$
- Evolve PDFs to any other scale $Q^2$
- Minimize error between theory and data
- Produce value of PDF at desired $(x, Q^2)$

Let’s look at some PDFs from the CTEQ collaboration
Parton distribution functions

$Q^2 = 1.69 \text{ GeV}^2$

$Q^2 = 100.00 \text{ GeV}^2$

$Q^2 = 1000.00 \text{ GeV}^2$

$Q^2 = 10000.00 \text{ GeV}^2$
Factorization and semi-inclusive DIS

Suppose a particular hadron $h$ is wanted in the final state of a process, such as $pp \rightarrow hX$

Factorize the cross section:

$$
\frac{d\sigma_{pp\rightarrow hX}}{dk_i dk_j} = \sum_{f_1, f_2, f} \int dx_1 \, dx_2 \, dy \, q_{f_1}(x_1, \mu^2) q_{f_2}(x_1, \mu^2) \left( \frac{d\hat{\sigma}_{f_1 f_2 \rightarrow f X'}}{dk_i dk_j} \right) D_f^h(z, \mu^2)
$$

Required inputs:

- universal parton distributions $q_{f_1}(x_1, \mu^2), q_{f_2}(x_1, \mu^2)$
- partonic cross section $d\hat{\sigma}_{f_1 f_2 \rightarrow f X'}/dk_i dk_j$
- fragmentation function $D_f^h(z, \mu^2)$
Conclusion

- The proton’s structure can be described using PDFs
- PDFs can be probed using DIS
- Gluons and sea quarks, not just valence quarks, make significant contributions to the proton’s momentum
- QCD can be used to evolve PDFs from one energy scale to another
- PDFs play an important role in nuclear and particle physics
  - understanding structure
  - input to other experiments
References

1. K. Nakamura et al. (PDG), JP G 37, 075021 (2011)
2. George Sterman et al. (CTEQ), Handbook of perturbative QCD (2001)
Conclusion

Thank you
Spin structure of proton

Polarized scattering cross section depends on polarized structure functions

\[ g_1 \propto \sum_f (\Delta q_f + \Delta \bar{q}_f) \]

\[ g_5 \propto \sum_f (\Delta q_f - \Delta \bar{q}_f) \]

where the polarized PDFs are given by \( \Delta q = q^{↑↑} - q^{↑↓} \)

Experiment shows that quark polarization carries less than half of the proton spin...where is the rest?

- orbital angular momentum
- gluon polarization \( \Delta g \) (RHIC)
SLAC-MIT

- First DIS experiments began in 1968
- $e^-$ on hydrogen
- $E_e = 7 - 17$ GeV (eventually 21 GeV)
HERA

Hadron Electron Ring Accelerator (DESY, Hamburg, Germany)

- $e^- p$ collider
- $E_e \approx 27$ GeV, $E_p \approx 820$ GeV $\rightarrow \sqrt{s} = 300$ GeV

H1          ZEUS
CERN

- $\mu^+$ on liquid hydrogen target
- $E_{\mu}^{\text{max}} \approx 280$ GeV, $M_p = 938$ MeV $\rightarrow \max \sqrt{s} = 22.9$ GeV
- Three notable collaborations
  - BCDMS (Bologna-CERN-Dubna-Munich-Saclay)
  - EMC (European Muon Collaboration)
  - NMC (New Muon Collaboration)
Fermilab

E665

• $\mu^+$ on liquid hydrogen target
• $\langle E_\mu \rangle = 470$ GeV $\rightarrow \langle \sqrt{s} \rangle = 29.7$ GeV
• Overlap in $x$ and $Q^2$ with NMC, HERA

NuTeV

• Separate $\nu_\mu$, $\bar{\nu}_\mu$ beams on steel (690 tons!)
• $E_\nu = 30 - 500$ GeV