Quarks inside Nucleons: Distribution and nuclear effects

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Graduate Seminar
OUTLINE

Deep Inelastic Scattering
   The Process
   Structure Functions and Parton Model

Parton Distribution Functions
   Extraction of PDF from Cross Section
   Valence and Sea quark
   Necessity of the Gluon

Drell-Yan Process
   The Processes
   Some results: light sea-quark Asymmetry

Effects of the Nuclear Medium
   Explanations
HADRONS AND QUARKS

Hadrons are particles that can interact via strong interaction. They are not fundamental particles since they are made by Quarks. The reason is the fact that excited states were observed.
What ‘Made by’ Means

Baryon and Meson Octets

\[ s = 0 \]
\[ n \quad p \]
\[ s = -1 \]
\[ \Sigma^- \quad \Sigma^0 \quad \Sigma^+ \quad \Lambda \]
\[ s = -2 \]
\[ \Xi^- \quad \Xi^0 \]
\[ q = -1 \quad q = 0 \]

\[ s = 1 \]
\[ K^0 \quad K^+ \]
\[ s = 0 \]
\[ \pi^- \quad \pi^0 \quad \pi^+ \quad \eta \]
\[ s = -1 \]
\[ K^- \quad \bar{K}^0 \]
\[ q = -1 \quad q = 0 \]

It is possible to classify Baryon and Meson in Octets according to their quantum numbers, showing this symmetry
**What ‘Made by’ Means**

The ‘Constituents’ of hadrons should be able to preserve this symmetry. This is the way how quarks up \( u \), down \( d \) and strange \( s \) were introduced; their quantum numbers had to match the hadron they are part of.

\[
\begin{align*}
p &\leftrightarrow uud \\
n &\leftrightarrow udd \\
\Lambda &\leftrightarrow uds \\
\Sigma^+ &\leftrightarrow uus
\end{align*}
\]

Features: \( q = 2/3 \) or \(-1/3\), \( \sigma = 1/2 \)…
How do we explore the distribution of Quarks?
Deep Inelastic Scattering
The typical size of a nucleon is $\sim 1$ fm ($10^{-15}$ m). We need a very small probe to explore its inner structure, therefore high energy leptons are used, with $E \gg 1$ GeV, such that

$$\lambda \sim \frac{1}{E} < 0.2 \text{ fm}$$

The process is called Deep Inelastic Scattering.
Deep Inelastic Scattering

The cross section for this process can be written by using Feynman diagrams.
The transition matrix element is

\[ iM = (-ie)^2 \left( \frac{-i}{q^2} \right) \langle k', s'_\ell | j_{\ell}^\mu (0) | k, s_\ell \rangle \langle X | j_{h\mu} (0) | p, s \rangle \]  

(2)

\( j_{h\mu} \) and \( j_{\ell\mu} \) are called hadronic and leptonic currents.

At this point it is useful to introduce the tensors \( \ell_{\mu\nu} \) and \( W^{\mu\nu}(p, q) \), again, leptonic and hadronic.
Deep Inelastic Scattering

\[ \ell^{\mu\nu} = \frac{1}{2} \sum_{s_{\ell}, s'_{\ell}} \langle k', s'_{\ell} \ | j^\nu_{\ell} (0) \ | k, s_{\ell} \rangle \langle k, s_{\ell} \ | j^\mu_{\ell} (0) \ | k', s'_{\ell} \rangle \] (3)

\[ W^{\mu\nu} = \frac{1}{4\pi} \frac{1}{2} \sum_s \int d^4 y e^{i q \cdot y} \langle p, s \ | [j^\mu (y), j^\nu (0)] \ | p, s \rangle \] (4)

The cross section can be related to these tensors \( (Q^2 = -q^2) \)

\[ \frac{d^2 \sigma}{dE' d\Omega} = \frac{e^4}{16\pi^2 Q^4} \left( \frac{E'}{ME} \right) \ell^{\mu\nu} W^{\mu\nu}(p, q) \] (5)

Source: Manohar 1992
The leptonic tensor is known, once we have solved Dirac equation for the leptons; evaluating the Feynmann diagram at the photon/lepton vertex we get

\[
\ell_{\mu\nu} = \frac{1}{2} \sum_{s_\ell, s'_\ell} (\bar{u}(k', s'_\ell)\gamma^\mu u(k, s_\ell))^* (\bar{u}(k', s'_\ell)\gamma^\nu u(k, s_\ell)) 
\]  

This expression can be simply reduced for unpolarized scattering by considering the properties of the Dirac spinors, to find

\[
\ell_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - \eta_{\mu\nu}(k \cdot k' - m^2) 
\]
STRUCTURE FUNCTIONS

While the hadronic can be only parametrized. We use the Structure Functions

\[ W_{\mu\nu} = F_1(x, q^2) \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{2x}{q^2} F_2(x, q^2) \left( p_\mu + \frac{q_\mu}{2x} \right) \left( p_\nu + \frac{q_\nu}{2x} \right) \]

Where we have introduced the Bjorken \( x \) variable

\[ x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2m(E - E')} \]

We need a model to extract information from scattering data
**Structure Functions**

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(8)

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**Parton Model (Feynman 1969)**

Features:

- Non-interacting *partons*
- one-to-one elastic scattering
- $x$ is the fraction of the total momentum
- Structure function depend on $x$ only (*Scaling*)
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**Parton Model**

\( F_1(x) \) and \( F_2(x) \) are related to the *parton distribution functions* (PDF)

\[
F_1(x) = \sum_i \frac{e_i^2}{2} f_i(x) \tag{10}
\]

\[
F_2(x) = \sum_i x e_i^2 f_i(x) \tag{11}
\]

Callan-Gross relation

\[
F_2(x) = 2xF_1(x) \tag{12}
\]

Source: Manohar 1992
**Parton ↔ Quark?**

How can this model be reliable for *Strong Interaction* if it assumes no interaction at all?

*Answer:* *Running Coupling Constant* and *Asymptotic Freedom*

\[
\alpha_s(Q^2) = \frac{12\pi\alpha_0}{1 + (33 - 2N_f)\ln\left(\frac{Q^2}{\Lambda^2}\right)}
\]  

(13)
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Source: Greiner 1994
**Running Coupling**

The QCD coupling as measured in physics processes at different scales $Q$, together with the band obtained by running the world average for $\alpha_s$ within its uncertainties. Figure taken from Ref. [17].

At hadron colliders, the best available complete calculation (i.e., all diagrams at a given order), doesn't go beyond $\alpha_s^2$ or $\alpha_s^3$. Certain subsets of diagrams (e.g., those without loops) can be calculated up to $\alpha_s^\infty$. So we are faced with a problem. Exact lattice methods can't deal with the high momentum scales that matter, exact perturbative methods can't deal with low-momentum scales that inevitably enter the problem, nor the high multiplicities that events have in practice. Yet, it turns out that we are reasonably successful in making predictions for collider events. These lectures will try to give you an understanding of the methods and approximations that are used.

2. Considering $e^+e^-\rightarrow$ hadrons

One simple context in which QCD has been extensively studied over the past 30 years is that of $e^+e^-\rightarrow$ hadrons. This process has the theoretical advantage that only the final state involves QCD. Additionally, huge quantities of data have been collected at quite a number of colliders, including millions of events at the $Z$ mass at LEP and SLC. We therefore start our investigation of the properties of QCD by considering this process.

2.1 Soft and collinear limits

There is one QCD approximation that we will repeatedly make use of, and that is the soft and collinear approximation. 'Soft' implies that an emitted gluon has very little energy compared to the parton (quark or gluon) that emitted it. 'Collinear' means that it is emitted very close in angle to another parton in the event. By considering gluons that are soft and/or collinear one can drastically simplify certain QCD calculations, while still retaining much of the physics.

The soft and collinear approximation is sufficiently important that it's worth, at least once, carrying out a calculation with it, and we'll do that in the context of the emission of a gluon from $e^+e^-\rightarrow q\bar{q}$ events. Though there are quite a few equations in the page that follows, the manipulations are all quite simple! We're interested in the hadronic side of the $e^+e^-\rightarrow q\bar{q}$ amplitude, so let's first write the QED matrix element for a virtual photon $\gamma^* \rightarrow q\bar{q}$ (we can always put back the $e^+e^-\rightarrow \gamma^*$ and the photon $\gamma$).
PARTON DISTRIBUTION FUNCTIONS
The Factorization Theorem

To extract the Parton Distribution Functions (PDF) within the parton model we need a useful theorem: The factorization theorem, that states:

\[ \sigma_{eH}(x, Q^2) = \sum_a \int_x^1 d\xi \ f_{a/H}(\xi) \sigma_B(x/\xi, Q^2) \]

(14)

Where \( \sigma_B(x, Q^2) \) is the Born cross section for the QED process \( eq \to eq \) at leading order.
Not only $d(x)$ and $u(x)$ in the proton. This is not true: Vacuum fluctuations imply $q\bar{q}$ pair formation.

\[
q_V(x) \equiv q(x) - \bar{q}(x)
\]
\[
q_S(x) \equiv 2\bar{q}(x)
\]
\[
\int dx (u(x) - \bar{u}(x)) = 2
\]
\[
\int dx (d(x) - \bar{d}(x)) = 1
\]
\[
\int dx (s(x) - \bar{s}(x)) = 0
\]
**SUM RULE**

Since $x$ is the fraction of the momentum carried by a parton we expect for the integral

$$
\sum_a \int_0^1 dx \, x \, q_a(x) = 1
$$

(15)
**SUM RULE**

Experimental results are quite different

\[ \sum_a \int_0^1 x q_a(x) = ? \]  \hspace{1cm} (16)

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Source: Salam 2011
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Source: Salam 2011
Deep Inelastic Scattering

Parton Distribution Functions

Drell-Yan Process

Effects of the Nuclear Medium

What are we missing?

The Gluon

\[ xq(x), \, xg(x) \]

\[ Q^2 = 10 \text{ GeV}^2 \]

CTEQ6D fit

\[ \text{gluon} \]

\[ d_S, \, u_S, \, u_Y \]

Source: Salam 2011
What are we missing?

The Gluon

\[ x_\nu(x), x_g(x) \]

\[ Q^2 = 10 \text{ GeV}^2 \]

CTEQ6D fit

gluon

d, u, s

Source: Salam 2011
Drell-Yan Process
Drell-Yan process

When talking about Drell-Yan process we refer to a bunch of possible processes that take place when hadrons collide. These are:

\[ h_1 + h_2 \rightarrow \mu^+ + \mu^- + X \]
\[ h_1 + h_2 \rightarrow e^+ + e^- + X \]
\[ h_1 + h_2 \rightarrow W^\pm + X \]
\[ h_1 + h_2 \rightarrow Z^0 + X \]
Drell-Yan process

The common thing among these processes is the fact that the basic process is the interaction between two quarks:

\[ q\bar{q}' \rightarrow \text{something} \]

The kind of interaction depends on the flavour of the interacting quarks:

- The first two QED processes
- The second two are obviously Weak processes, since flavour change is needed
**Drell-Yan process**

\[
\begin{align*}
q &\rightarrow \ell^+ \\
\bar{q} &\rightarrow \ell^-
\end{align*}
\]
Drell-Yan process

\[ q_a \bar{q}_b W^{\pm} \]

\[ h_1 \quad h'_1 \quad h_2 \quad h'_2 \]
**Drell-Yan process**

Some calculations:

\[ p_1 = x_1 P_1, \quad p_1 = x_2 P_2 \]

\[
d\sigma = \frac{1}{2\hat{s}} |\mathcal{M}|^2 d\Gamma
\]  

(17)

\[
i\mathcal{M} = iQ_f \frac{e^2}{k^2} (\bar{u}'(p_2)\gamma^\mu u^s(p_1)) (\bar{u}(p_3)\gamma_\mu v'^r(p_4))
\]

(18)

\[
\sigma(q_f \bar{q}_f \rightarrow \ell\bar{\ell}) = \frac{1}{3} Q_f^2 \frac{4\pi\alpha^2}{3\hat{s}}
\]

(19)

Source: Peskin 1995
**Drell-Yan process**

Some calculations:

\[
\frac{d^2\sigma}{dx_1dx_2} = \frac{4\pi\alpha^2}{9M^2} \sum_f Q_f^2 \left[ q_f(x_1)\bar{q}_f(x_2) + \bar{q}_f(x_1)q_f(x_2) \right] \tag{20}
\]

So it is possible to extract experimentally direct information about the anti-quark distribution in the proton.

Source: Peskin 1995
SOME RESULTS
It is therefore possible to calculate these distribution via $pp$ and $pn$ scattering (using deuterium target)

\[ \sigma^{pp} \propto \frac{4}{9} u_p(x_1) \bar{u}_p(x_2) + \frac{1}{9} d_p(x_1) \bar{d}_p(x_2) \]  

(21)

\[ \sigma^{pn} \propto \frac{4}{9} u_p(x_1) \bar{u}_n(x_2) + \frac{1}{9} d_p(x_1) \bar{d}_n(x_2) \]

\[ = \frac{4}{9} u_p(x_1) \bar{d}_p(x_2) + \frac{1}{9} d_p(x_1) \bar{u}_p(x_2) \]  

(22)

Where the last equality was obtained by using isospin symmetry

\[ \bar{u}_n(x) \rightarrow \bar{d}_p(x) \]

\[ \bar{d}_n(x) \rightarrow \bar{u}_p(x) \]  

(23)
Some results

NuSea Collaboration data show a sensitive asymmetry in the light sea quark component, that cannot be explained with perturbative QCD.

\[
\frac{\sigma^{pd}}{2\sigma^{pp}} \bigg|_{x_1 \gg x_2} \approx \frac{1}{2} \left[ 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \right]
\] (24)
Effects of the nuclear medium
**Nuclear Effects**

When the target is a nucleus the Structure Function are modified. The ratio $R_A = F_2^A / F_2^D$ has the following behaviour:

![Graph showing the behavior of $F_2^A / F_2^D$ with $x$ on the x-axis and $F_2^A / F_2^D$ on the y-axis. The graph shows the transition from shadowing to antishadowing to Fermi motion.]

Source: Arneodo 1992
**Some Explanations**

- **Shadowing and Anti-Shadowing**
  
  Virtual photon fluctuation into meson state → Shadow effect at small $x$;

- **Fermi motion**
  
  Nucleons are not stationary but move with average momentum $k_F$;

- **EMC effect**
  
  Only phenomenological models, not an actual theory.
CONCLUSION

▶ The composition of hadrons is much more complicated than naive approach
▶ DIS allows to extract information about PDF, fundamental to control heavy ion scattering
▶ Drell-Yan process is powerful to extract the sea quark component of PDF
▶ When nucleons are bound together the distribution of quarks gets modified, need to understand causes and consequences
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G. P. Salam, CERN Yellow Report CERN-2011-002, 45-100


R. S. Towell et al. [FNAL E866/NuSea Collaboration], Phys. Rev. D 64 (2001) 052002


THANKS
EXTRA

- QCD
- Nuclear effects regions
- Perturbative QCD
- Scaling
- Gluon Splitting
- Overlap
- DGLAP