1. Introduction

2. The Special Theory of Relativity
   - departures from Newtonian dynamics
   - the nature of light
   - the Michelson-Morley experiment

Textbook: A.P. French “Special Relativity”, chapter 1,2,3 (selected subsections – see next page)
Lecture 4 covers:
A.P. French “Special Relativity”

Chapter 1 Departures from newtonian dynamics
- Newton
- “The ultimate speed”

Chapter 2 Perplexities in the propagation of light
- The nature of light
- The luminiferous ether
- Prelude to the Michelson-Morley experiment
- The Michelson-Morley experiment
- Concluding remarks

Chapter 3
- Relativity according to Galileo and Newton
- The transformations of Newton’s law
Modern Physics

- A survey of the major physics theories of the 20th century (relativity and quantum mechanics) and their impact on most areas of physics.

- It introduces the special theory of relativity, the concepts of quantum and wave-particle duality, Schroedinger's wave equation, and other fundamentals of quantum theory as they apply to nuclei, atoms, molecules, and solids.
Following the initial success of 1911, the Solvay Conferences have been devoted to outstanding preeminent open problems in both physics and chemistry.

Micro-world
Small sizes/distances

SI – International System of Units

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<thead>
<tr>
<th>Unit</th>
<th>Length</th>
<th>Prefix</th>
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<tr>
<td>mm</td>
<td>$10^{-3}$ m</td>
<td>milli-</td>
</tr>
<tr>
<td>µm</td>
<td>$10^{-6}$ m</td>
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<tr>
<td>nm</td>
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<td>femto-</td>
</tr>
<tr>
<td>am</td>
<td>$10^{-18}$ m</td>
<td>atto-</td>
</tr>
</tbody>
</table>

Less than $0.01$ m
- crystal

$10^{-9}$ m
- molecule

$10^{-10}$ m
- atom

$10^{-14}$ m
- nucleus

$10^{-15}$ m
- nucleon

Less than $10^{-18}$ m
- leptons (e.g. electron), quarks (fermions)

understood to be point-like particles
Energy Units

Energy is defined via work. The SI unit for energy is joule J:

\[ J = \frac{kg \cdot m^2}{s^2} = N \cdot m \]

**Electronvolt (eV)** - an energy unit that is used in atomic physics, particle physics and high energy physics.

\[ 1 \text{ eV} = \text{the amount of kinetic energy gained by a single unbound electron when it accelerates through an electric potential difference of 1 Volt (1 J per Coulomb) multiplied by the electron charge (1 e, or 1.60217653(14) \times 10^{-19} C).} \]

\[ E = qV \quad \Rightarrow \quad 1 \text{ eV} = 1.60217653(14) \times 10^{-19} J \]

- **eV is NOT an SI unit and its value must be obtained experimentally**
- It is commonly used with the SI prefixes milli-, kilo-, mega-, giga-, tera-, or peta-(meV, keV, MeV, GeV, TeV and PeV respectively)

Energy Units

\[ 1 \text{ keV} = 10^3 \text{ eV} \]

13.6 eV

Ionization energy of the hydrogen atom

What about mass?

Mass:

- Proton = \(1.6726231 \times 10^{-27} \text{ kg}\)
- Electron = \(9.1093897 \times 10^{-31} \text{ kg}\)

Tiny! inconvenient to use
Energy Units

1 keV = 10^3 eV
1 MeV = 10^6 eV
1 GeV = 10^9 eV

13.6 eV
0.511 MeV/c^2
0.938 GeV/c^2

Ionization energy of the hydrogen atom
Electron mass
Proton mass

\[ E = mc^2 \implies m = \frac{E}{c^2} \]

mass unit: eV/c^2

A. Einstein’s mass–energy equivalence formula \( E = mc^2 \) has been dubbed "the world's most famous equation"
Energy Units

1 keV = $10^3$ eV
1 MeV = $10^6$ eV
1 GeV = $10^9$ eV
1 TeV = $10^{12}$ eV
1 PeV = $10^{15}$ eV
1 EeV = $10^{18}$ eV
1 ZeV = $10^{21}$ eV

13.6 eV
0.511 MeV/$c^2$
0.938 GeV/$c^2$

Ionization energy
of the hydrogen atom
Electron mass
Proton mass
 Cosmic rays: highest observed energies
(natural accelerators)
Particle accelerators

A particle accelerator - a device that uses electromagnetic fields to propel electrically charged particles to high speeds / high energies.

There are two types of accelerators:

- **natural cosmic accelerator** – the Universe
  cosmic rays acceleration (possibly Active Galactive Nuclei)

- **man-made accelerators**
  particle accelerators, e.g. Relativistic Heavy Ion Collider (RHIC) at BNL, Large Hadron Collider (LHC) at CERN

Such devices wouldn’t work if designed according to Newtonian mechanics (practical implications of special relativity)
Simplification! Speed

Far less than $3 \times 10^8$ m/s

Comparable to $3 \times 10^8$ m/s

Constant: speed of light $c$

Classical Mechanics

Relativistic Mechanics

This course

Planck constant $h$

Quantum Mechanics

Quantum Field Theory

This course
Newton's laws of motion

- **First law:** \[ \sum F = 0 \implies \frac{dv}{dt} = 0. \]

Every object continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by external forces acted upon it.

- **Second law:** \[ F = m \frac{dv}{dt} = ma, \]

The acceleration \( a \) of a body is parallel and directly proportional to the net force \( F \) acting on the body, is in the direction of the net force, and is inversely proportional to the mass \( m \) of the body. **Assumption:** \( m = \text{const} \)

- **Third law:** \[ \sum F_{a,b} = - \sum F_{b,a} \]

When two bodies interact by exerting force on each other, these action and reaction forces are equal in magnitude, but opposite in direction.
To describe an event one needs a reference frame.

Newton laws are valid in *inertial frames of reference* (a reference frame in which an object subjected to no forces moves in a straight line at constant speed).

*The principle of Newtonian relativity:* the laws of mechanics must be the same in all inertial frames of reference.

All observers are equivalent and the laws of nature must take the same mechanical form for all observers (Newton’s laws and conservation of energy, momentum).
Principle of Newton Relativity

Newton laws are valid in *inertial frames of reference*.

The experiment looks different in different inertial frames (laboratory, moving truck), and the observers measure different values of position and velocity of the ball at the same times, both observers agree on the validity of Newton’s law, conservation of energy and momentum.

No experiment involving mechanics can detect any difference between the two inertial frames.
Equivalence of measurements made in different reference frames:

- the equivalence of different frames for doing physics
- we need a mathematical formula (transformation) that systematically relates observations made in one reference frame to another.

Galilean transformation – such that Newtonian relativity is satisfied.

Any force is invariant under the Galilean transformations as long as it involves the relative positions of interacting particles.
Galilean transformation of coordinates (1 dimension)

\[ \begin{align*}
\mathbf{v} & \quad \mathbf{v} \\
\mathbf{x} & \quad \mathbf{x}' \\
\mathbf{y'} & \quad \mathbf{y} \\
\mathbf{z'} & \quad \mathbf{z} \\
\mathbf{t'} & \quad \mathbf{t}
\end{align*} \]

\[ \begin{align*}
x' &= x - vt \\
y' &= y \\
z' &= z \\
t' &= t
\end{align*} \]

In classical mechanics, all clocks run at the same rate, regardless of their speed (classical mechanics breaks down as \( v \sim c \))

\[ \frac{d}{dt'} x' = \frac{d}{dt} x - v \quad \Rightarrow \quad u_x' = u_x - v \]

Galilean addition law for velocities

\[ \frac{d^2}{dt^2} x' = \frac{d^2}{dt^2} x - \frac{d}{dt} v \Rightarrow a_x' = a_x \]

Newton’s laws are the same in the moving system!
Galilean transformation of coordinates (1 dimension)

\[ u_x' = u_x - v \]  

speed of light measured inside of a moving car

\[ v = 200 000 \text{ km/s} \]
\[ = 2 \times 10^8 \text{ m/s} \]

By measuring the speed of light going past the car (if Galilean transformation is correct for light) one could determine the speed of the car.

Several experiments to determine the velocity of the earth were based on this general idea.

\[ u_x' = 3 \times 10^8 \text{ m/s} - 2 \times 10^8 \text{ m/s} = 1 \times 10^8 \text{ m/s} < c \]
Galilean transformation of coordinates (1 dimension)

\[ u'_x = u_x - v \]

speed of light measured inside of a moving car

\[ v = 200 \, 000 \, \text{km/s} = 2 \times 10^8 \, \text{m/s} \]

Experiment has proven otherwise

\[ u'_x = 3 \times 10^8 \, \text{m/s} - 2 \times 10^8 \, \text{m/s} = 1 \times 10^8 \, \text{m/s} < c \]

- By measuring the speed of light going past the car (if Galilean transformation is correct for light) one could determine the speed of the car.
- Several experiments to determine the velocity of the earth were based on this general idea.
Newton’s second law’s consequence:

\[ F = m \frac{dv}{dt} = ma, \]

- no upper limit on velocity

\[ (v_{\text{max}})^2 = c^2 \]

Newtonian prediction

\[ E_{\text{kin}} = \frac{mv^2}{2} \]

Measured!
The Speed of Light

- central to our **modern** ideas about space and time.

“It is now believed that this speed of \( c \) is a limit that can never be reached or exceeded by particles with non-zero rest mass and it is the greatest speed at which information can pass from one place to another.”

S. Adams, “Frontiers Twentieth century physics”
Stock trades to exploit speed of light, says researcher

By Jason Palmer
Science and technology reporter, BBC News, Dallas

Financial institutions may soon change what they trade or where they do their trading because of the speed of light.

“High-frequency trading” carried out by computers often depends on differing prices of a financial instrument in two geographically-separated markets.

Exactly how far the signals have to go can make a difference in such trades.

Or more recent:
http://www.nature.com/polopoly_fs/1.16872!/menu/main/topColumns/topLeftColumn/pdf/518161a.pdf
The Speed of Light
- first measurements
"First measurement by Roemer, who observed that times of eclipses of the moons of Jupiter had a systematic variation in addition to their period of orbit that seemed to be linked to the relative positions of Jupiter and the Earth in their respective orbits. In 1679 he predicted that a certain eclipse will be ~10 min later than expected (assuming a fixed period) because of a delay in the light reaching the Earth as it crosses the Earth’s orbit."

Roemer result: \( c = 2 \times 10^8 \text{m/s} \)

S. Adams, “Frontiers Twentieth century physics”
The Speed of Light

\[ c \approx 3 \times 10^8 [m/s] \]
\[ d \approx 150 \times 10^6 [km] \]
\[ t \approx 8 [min] \]
The Nature of Light
The propagation of light involves the transport of energy away from the source:

- **Particle theory of light**
  - 6th century B.C., Pythagoras: stream of particles emitted from the source (light propagates in straight lines and can travel through vacuum)
  - 20th century, A. Einstein: Compton effect

- **Wave theory of light**
  - 17th century, R. Hook: light is a vibration communicated through a medium of some kind; Huygens showed that wave theory could account for reflection and refraction.
  - 19th century: diffraction, interference, polarization

If light is a wave, what’s the medium?
The speed of the solar system through the ether
Clerk Maxwell, 1879

Is the whole solar system moving through the ether with some speed v?
Speed of light (classic, Galilean transformation)

- **Downwind**
  \[ v + c \]

- **Upwind**
  \[ c - v \]

- **Across**
  \[ \sqrt{c^2 - v^2} \]
The speed of the solar system through the ether
Clerk Maxwell, 1879

If the times of eclipses are measured when Jupiter is at first at A’ and then, 6 years later at B’, we hope to discover whether the whole solar system is moving through the ether with some speed $v$. 

$T_{Jupiter} \sim 12 \ T_{Earth}$
The Michelson-Morley Experiment (1887)

Goal: determine the absolute velocity of the Earth through the hypothetical “ether”.

Why is this important? The existence of ether and a preferred ether frame would show that light was similar to other classical waves in requiring a medium (e.g. sound waves).

Result: null result (no difference was found in the observed speed of light in any direction), explained 18 years later by A. Einstein.
The Michelson-Morley Experiment (1887)

http://video.mit.edu/watch/michelson-interferometer-6561/
The Michelson-Morley Experiment (1887)

http://video.mit.edu/watch/michelson-interferometer-6561/
The Michelson-Morley Experiment (1887)

The total time of travel for the round trip:

- **PM₁ and M₁P:**

  \[ t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v} = \frac{2l_1}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-1} \]

- **PM₂ and M₂P:**

  \[ t_2 = \frac{2l_2}{\sqrt{c^2-v^2}} = \frac{2l_2}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \]

  (since the speed of the beam relative to the Earth \( \sqrt{c^2-v^2} \))
Taylor series expansion (will be covered in Recitation 3)

$$f(x) = f(x_0) + \frac{df}{dx}\bigg|_{x=x_0} (x-x_0) + \frac{1}{2!} \frac{d^2 f}{dx^2}\bigg|_{x=x_0} (x-x_0)^2 + \ldots$$

$x \ "very \ small"$

$x \to 0$

$x << 1$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots$$

$$\frac{1}{1+x} = 1 - x + x^2 + \ldots$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots$$

$$\alpha = \text{const}$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \ldots$$

http://en.wikipedia.org/wiki/Taylor_series
The Michelson-Morley Experiment (1887)

The time difference between the light travelling horizontally and vertically

\[
\Delta t = t_1 - t_2 = \frac{2l_1}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-1} - \frac{2l_2}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}
\]

\approx \frac{2l_1}{c} \times \left(1 + \frac{v^2}{c^2}\right) - \frac{2l_2}{c} \times \left(1 + \frac{v^2}{2c^2}\right)

\Delta t \approx \frac{2(l_1 - l_2)}{c} + \frac{2l_1v^2}{c^3} - \frac{l_2v^2}{c^3}

- If \( \Delta t = 0 \) then emerging beams will be in phase and will reinforce each other.
- If \( \Delta t \neq 0 \) the beams will be out of phase and interference will result.

if \( l_1 = l_2 \) and \( v=0 \) \( \Rightarrow \Delta t = 0 \)

if \( l_1 = l_2 \) and \( v \neq 0 \) \( \Rightarrow \Delta t \neq 0 \)
The Michelson-Morley Experiment (1887)

The time difference between the light travelling horizontally and vertically

\[ \Delta t = t_1 - t_2 = \frac{2l_1}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-1} - \frac{2l_2}{c} \times \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \]

\[ \approx \frac{2l_1}{c} \times \left(1 + \frac{v^2}{c^2}\right) - \frac{2l_2}{c} \times \left(1 + \frac{v^2}{2c^2}\right) \]

\[ v \ll c \]

\[ \Delta t \approx \frac{2(l_1 - l_2)}{c} + \frac{2l_1v^2}{c^3} - \frac{l_2v^2}{c^3} \]

The time difference between the light travelling horizontally and vertically

After we rotate the interferometer by 90°

\[ \Delta t' = t'_1 - t'_2 \approx \frac{2l_1}{c} \times \left(1 + \frac{v^2}{2c^2}\right) - \frac{2l_2}{c} \times \left(1 + \frac{v^2}{c^2}\right) \]

\[ \Delta t' \approx \frac{2(l_1 - l_2)}{c} + \frac{l_1v^2}{c^3} - \frac{2l_2v^2}{c^3} \]
The Michelson-Morley Experiment (1887)

The change of time difference would lead to a shift of the interference pattern by an amount corresponding to $\delta$ fringes:

$$\delta = \frac{c(\Delta t - \Delta t')}{\lambda} = \frac{(l_1 + l_2)v^2}{\lambda c^2}$$

if $l_1 = l_2 = l \Rightarrow \delta = \frac{2(v/c)^2}{\lambda/l}$

if $v \approx 30 \, \text{km/s} \Rightarrow \frac{v}{c} \approx 10^{-4}$

if $\lambda \approx 6 \times 10^{-7} \, \text{m}, \ l = 1.2 \, \text{m} \Rightarrow \frac{\lambda}{l} \approx 5 \times 10^{-7}$

$\Rightarrow \delta \approx 0.04 \, \text{fringe}$

“*The result of the hypothesis of a stationary ether is thus shown incorrect*”

- A.A. Michelson
### TRIALS OF THE MICHELSON-MORLEY EXPERIMENT

<table>
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<tr>
<th>Observer; year</th>
<th>$l$, cm.</th>
<th>$\delta_{\text{calc}}$</th>
<th>$\delta_{\text{obs}}$ (upper limit)</th>
<th>Ratio</th>
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<td>120</td>
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*From a review by Shankland et al., Rev. Mod. Phys., 27, 167 (1955).*
Michelson-Morley experiment result (by some accounts a failure) means that there is no ether.

This means that light does not have a special medium with which it propagates.

http://www.nature.com/polopoly_fs/1.16872!/menu/main/topColumns/topLeftColumn/pdf/518161a.pdf