An electron of energy $E$ ($E < V_0$) is incident upon a rectangular barrier of height $V_0 = 10$ eV and thickness $a = 1.8 \times 10^{-10}$ m, as shown below:

This rectangular barrier is an idealization of the barrier encountered by an electron that is scattering from a negatively ionized gas atom in the "plasma" of a gas discharge tube. The actual barrier is not rectangular, of course, but it is about the height and thickness quoted.

a) Using the Schroedinger equation, show that the transmission coefficient $T$ (defined as a ratio of the probability flux transmitted through the barrier into the region $x > a$, to the probability flux incident upon the barrier) is:
b) Calculate the transmission coefficient and the reflection coefficient $R$, assuming the total energy $E$ of the electron is i) $V_0/2$ and ii) $V_0/100$. How much would these coefficients change if the potential barrier width “$a$” was reduced by a factor of 100. Discuss the results.

$$T = \frac{v_1 C^* C}{v_1 A^* A} = \left[ 1 + \frac{(e^{k_1 a} - e^{-k_1 a})^2}{16 \frac{E}{V_0} \left( 1 - \frac{E}{V_0}\right)} \right]^{-1} = \left[ 1 + \frac{\sinh^2 k_1 a}{4 \frac{E}{V_0} \left( 1 - \frac{E}{V_0}\right)} \right]^{-1}$$

where

$$k_1 a = \sqrt{\frac{2m V_0 a^2}{h^2}} \left( 1 - \frac{E}{V_0}\right), \quad E < V_0$$