1. (from the textbook)
A proton collides elastically with another proton at rest. For simplicity we shall limit ourselves to a special case in which, after collision, the two particles travel symmetrically at equal angles $\theta/2$ to the direction of the incident proton and with equal momenta. Find $\theta$ knowing the kinetic energy of the incident proton. Proton rest mass in known ($m_0$)

Result: 

$$\cos^2 \left( \frac{\theta}{2} \right) = \frac{E_{1}^{kin} + 2m_0c^2}{E_{1}^{kin} + 4m_0c^2}$$

STEP 1:
Let the incident proton have total energy $E_1$ and momentum $p_1$ (along x axis). Let the momenta of the of the particles after collision be of magnitude $p_2$ at angles +/- $\theta/2$ to the direction of $p_1$.  

![Fig. 6-9 Relativistic elastic collision of a particle with a similar particle initially at rest. The final state is assumed to be a symmetrical one in which the particles have equal speeds and hence make equal angles with the initial direction of particle 1.](image)
Additional problems (instead of this week’s homework)
Practice at home (part of midterm1 preparation)

2. A pion $\pi$ with rest mass $m_0^\pi$ decays into a muon $\mu$ with rest mass $m_0^\mu$ and a neutrino $\nu$ whose rest mass is negligible. $\pi$ is at rest.
Show that the kinetic energy of the muon is given by:
$$E_{kin}^\mu = \frac{\left(m_0^\pi - m_0^\mu\right)^2}{2m_0^\pi}c^2$$

3. A positron (energy $E_1$ and momentum $p_1$) and an electron (energy $E_2$ and momentum $p_2$) are produced in a pair creating process.
   The rest mass $m_0$ of the electron and the positron are the same, $m_0 = 0.511\text{MeV}/c^2$.
a) What is the velocity of the frame in which the pair has zero momentum?
b) What is the energy either particle has in the zero-momentum frame.
c) What is the magnitude of relative velocity between the particles (i.e. the velocity of one particle as seen by an observer attached to the other.)

4. Find the center of mass frame velocity $\vec{V}$ (velocity of the center of mass frame with respect to the lab frame) knowing $\vec{p}_{total}$ and $E_{total}$:
Assume: $\vec{V} = (V,0,0), \vec{p}_{total} = (p_{total},0,0)$
Hint: use Lorentz transformations