On the Determination of Oscillation Parameters from Neutrino Oscillation Experiments

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**Lepton Mixing Matrix**

- $\nu$ weak and mass eigenstates related by lepton mixing matrix.
- For three **massive** neutrinos:

$$U = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta CP} \\
-s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta CP} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta CP} & c_{13} s_{23} \\
-s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta CP} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i\delta CP} & c_{13} c_{23}
\end{pmatrix} P$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. For Majorana neutrinos

$$P = \begin{pmatrix}
e^{i\eta_1} & 0 & 0 \\
0 & e^{i\eta_2} & 0 \\
0 & 0 & 1
\end{pmatrix}$$

and for Dirac neutrinos $P = I$. 
Neutrino Oscillations in Vacuum

The weak eigenstates $\nu_\alpha$ produced in weak interaction are linear combination of the mass eigenstates $\nu_i$

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}^* |\nu_i\rangle$$

where $n$ is the number of light neutrino species and $U$ is the mixing matrix. Then it evolves as

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{n} U_{\alpha i}^* |\nu_i(t)\rangle$$

It is oscillated into the other flavor $\beta$ with the probability

$$P_{\alpha\beta} = |\langle \nu_\beta |\nu_\alpha(t)\rangle|^2 = \left| \sum_{i=1}^{n} \sum_{j=1}^{n} U_{\alpha i}^* U_{\beta j} \langle \nu_j |\nu_i(t)\rangle \right|^2$$
Neutrino Oscillations in Vacuum

- $|\nu\rangle$ is a plane wave $|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i(0)\rangle$
- If neutrinos are relativistic $E_i = \sqrt{p_i^2 + m_i^2} \approx p + \frac{m_i^2}{2E}$
- The transition probability

\[
P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i<j}^{n} \Re[U_{\alpha i} U^{*}_{\beta i} U^{*}_{\alpha j} U_{\beta j}] \sin^2 X_{ij} + 2 \sum_{i<j}^{n} \Im[U_{\alpha i} U^{*}_{\beta i} U^{*}_{\alpha j} U_{\beta j}] \sin 2X_{ij}
\]

where $X_{ij} = \frac{(m_i^2 - m_j^2)L}{4E} = 1.27 \frac{\Delta m^2_{ij}}{eV^2} \frac{L}{m/MeV}$
- Often need to consider matter effect (e.g. MSW effect)
Neutrino Mass Ordering

Two possible orderings: **Nomal Ordering** and **Inverted Ordering**.

Normal Ordering (NO): $m_1 < m_2 \ll m_3$
Inverted Ordering (IO): $m_3 \ll m_1 < m_2$
Neutrino Oscillation Experiments

By 2015 we have observed with high (or good) precision:

- Solar $\nu_e$ convert to $\nu_\mu/\nu_\tau$ (Cl, Ga, SK, SNO, Borexino)
- Reactor $\bar{\nu}_e$ disappear at $L \sim 200$ Km (KamLAND)
- Atmospheric $\nu_\mu$ & $\bar{\nu}_\mu$ disappear most likely to $\nu_\tau$ (SK,MINOS)
- Accelerator $\nu_\mu$ & $\bar{\nu}_\mu$ disappear at 250[700] Km (K2K,T2K, [NOvA], [MINOS])
- Some accel $\nu_\mu$ appear as $\nu_e$ at 250[700] Km (T2K, [MINOS])
- Reactor $\bar{\nu}_e$ disappear at 1 Km (D-Chooz, Daya-Bay, Reno)
Solar Neutrinos

- Solar neutrinos are produced in the thermonuclear interactions
- Two main sources: pp chain and CNO cycle

\[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e + \gamma \]
**Borexino Experiment**

- Borexino measures neutrino-electron elastic scattering

\[ \nu_x + e \rightarrow \nu_x + e \]

- Borexino fit \( N_b = n_{el} T_b \text{run} \sum_{\alpha} \int \frac{d\Phi_\alpha}{dE_\nu} P(\nu_e \rightarrow \nu_\alpha) \frac{d\sigma_\alpha}{dT_e} R_b(T_e, T_{b,\text{min}}, T_{b,\text{max}}) dE_\nu dT_e \)

![Graph showing neutrino events vs energy](attachment:image.png)

From Borexino Collaboration (2014)
T2K Experiment

- T2K produces mainly $\nu_\mu$ and $\bar{\nu}_\mu$ fluxes
- Both $\nu_\mu$ disappearance and $\nu_e$ appearance will be detected
The reconstructed energy spectral of $\nu_e$ and $\nu_\mu$ compared with data

MC assumed: $\sin^2 2\theta_{13} = 0.1$, $\sin^2 2\theta_{23} = 1.0$, $\delta_{CP} = 0$, $\Delta m^2_{23} = 2.4 \times 10^{-3}$eV$^2$, NO

From T2K Collaboration (2015)
First goal is to determine mass ordering.
But event rates also depend significantly on the value of $\delta_{CP}$.
Sensitivity (# of $\sigma$) for rejecting IO(NO) if NO(IO) true.

From Blennow et al (2014)
Global Analyses of Neutrino Oscillation Experiments

*For dashed curves short baseline data are not included.*
### Oscillation Parameters: Present Status 1σ (3σ)

<table>
<thead>
<tr>
<th></th>
<th>Normal Ordering</th>
<th>Inverted Ordering</th>
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<tbody>
<tr>
<td></td>
<td>bfp±1σ</td>
<td>3σ range</td>
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<tr>
<td>( \theta_{12} /° )</td>
<td>( 33.48^{+0.78}_{-0.75} )</td>
<td>31.29→35.91</td>
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<tr>
<td>( \theta_{23} /° )</td>
<td>( 42.3^{+3.0}_{-1.6} )</td>
<td>38.2→53.3</td>
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<tr>
<td>( \theta_{13} /° )</td>
<td>( 8.50^{+0.20}_{-0.21} )</td>
<td>7.85→9.10</td>
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<tr>
<td>( \delta_{\text{CP}} /° )</td>
<td>( 306^{+39}_{-70} )</td>
<td>0→360</td>
</tr>
<tr>
<td>( \Delta m^2_{21} / 10^{-5}\text{eV}^2 )</td>
<td>( 7.50^{+0.19}_{-0.17} )</td>
<td>7.02→8.09</td>
</tr>
<tr>
<td>( \Delta m^2_{31} / 10^{-5}\text{eV}^2 )</td>
<td>( 2.457^{+0.047}_{-0.047} )</td>
<td>2.317→2.607</td>
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\[ |U| = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.514 \rightarrow 0.580 & 0.137 \rightarrow 0.158 \\ 0.225 \rightarrow 0.517 & 0.441 \rightarrow 0.699 & 0.614 \rightarrow 0.793 \\ 0.246 \rightarrow 0.529 & 0.464 \rightarrow 0.713 & 0.590 \rightarrow 0.776 \end{pmatrix} \]
Summary

- Lepton mixing matrix and neutrino oscillation
- Neutrino oscillation experiments
  - Solar experiments (Borexino)
  - Atmospheric experiments
  - Accelerator experiments (T2K)
  - Reactor experiments
- Global analyses of neutrino oscillation experiments
  - Present fit of oscillation parameters