The status of QCD and electron-ion collisions

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Stony Brook
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A personal view on some of where we are, how we got here, and where an EIC might help take us.

I. Prehistory: looking inward and confirming QCD
II. Looking outward toward low-$x$, TMDs and polarized scattering
III. A few words on factorization and universality
IV. In conclusion: tasks for an EIC
I. Prehistory: looking inward and confirming QCD

For hadron colliders (Tevatron, RHIC, LHC) the in-states are governed by the “strong” interactions, first observed in nuclear forces.

QCD is the quark/gluon gauge theory built on color ($q_a = q_1^a q_2^a q_3^a$):

$$\mathcal{L}_{QCD} = \sum_q \sum_{a,b} \bar{q} i\gamma \cdot (\delta_{ab}\partial - g_s A_{ab}) + \delta_{ab}m_q q_b$$

$$-\frac{1}{4} F_{\mu\nu}^2 [A]$$

(global symmetry: Han & Nambu, statistics: Greenberg)
(Pati-Salam 1972, 3 … Bardeen, Fritzsch, Gell-Mann, Leutwyler, 1972,3)

In this Lagrangian only quarks and gluons appear, not protons, neutrons or nuclei. How can we do without them?

QCD is universally recognized as the correct theory of strong interactions. How? Partly, by asking new sets of questions, finding ways in which the partonic degrees of freedom could manifest themselves as accelerators moved to higher energy. The program for an EIC, and its design, will depend on identifying the best match of questions, new and old, adapted to new experimental capabilities.
Seeing QCD

- The confirmation of QCD runs deeply through its quantum properties.
- Renormalization in a nutshell: To address this problem: the amplitude for a quark to enter a ball of finite radius $cT$, and come out as a quark + gluon is infinite! Too many histories! We sidestep this problem as follows . . .

- Solution: we (re)define $g_s(\mu) \equiv$ the amplitude for $q \to q + g$ inside a “ball” of radius $cT \equiv hc/\mu$. We still can’t compute $g_s(\mu)$ but we can compute what happens when we change the radius just a little, $dg_s/d\mu$:

  \[
  g(h/T) = + + \frac{cT}{g(h/T)} =
  \]

  Calculate $\mu dg_s(\mu)/d\mu = -b_0 g^3$, with $b_0 = 11 - 2n_{\text{quarks}}/3$, and we get:

  \[
  \alpha_s(\mu') \equiv \frac{g_s^2(\mu)}{4\pi} = \frac{\alpha_s(\mu_0)}{1 + b_0 \frac{\alpha_s(\mu_0)}{4\pi} \ln \left( \frac{\mu^2}{\mu_0^2} \right)} \equiv \frac{4\pi}{b_0 \ln \left( \frac{\mu^2}{\Lambda_{\text{QCD}}^2} \right)}
  \]
• This is asymptotic freedom: (Gross-Wilczek, Politzer (1973-4), Georgi)

\[ g_s(\mu = \infty) \to 0 \]

• And we can use any \( \mu \) we like, so long as \( \alpha_s(\mu) \) is small.

• Asymptotic freedom is a big deal:

\[
\frac{\text{Asymptotic Freedom}}{\text{Quantum Chromodynamics}} = \frac{\text{Elliptical Orbits}}{\text{Newtonian Gravity}}
\]

• But still, it’s just a beginning, not an end.

For Newtonian gravity, the three-body problem is qualitatively harder than the two-body problem.

And for QCD, asymptotic freedom is far from enough . . . confinement; hadronization. Applications of weak coupling and perturbation theory are fleeting.
How to study a theory with confinement?

• The goal

\[
\frac{\text{Nuclear Physics}}{\text{QCD}} = \frac{\text{Chemistry}}{\text{QED}}
\]

• But can we

  – Study the particles that give the currents (quarks)?

  – Expand in number of gluons? Perturbation Theory: quantized “weak fields”.

  – The origin of the term “perturbation theory” puts our efforts in context.

  – We are forty-plus years from the discovery/invention/creation of QCD. Shouldn’t we know it all already?

  – Go back to the analogy to Newtonian gravity. Not much than forty years after 1686 ...
The lunar appogee in the eighteenth century

- Sep. 1747 (Euler to Clairaut): “...the forces that act on the moon do not follow the rule of Newton.”

- Nov. 1747 (Clairaut to French Royal Academy): Newton’s theory of graviation is ruled out

- June 1748 (d’Alembert to Cramer): “...the gravitation of the moon to the sun will not explain [the] irregularities of its motion.”

- May 1749 (Clairaut to French Royal Academy): Terms previously neglected show Newton’s law was right all along.

→ $1/r^4$ – new law (magnetism?) or correction?
→ theory of the three-body problem
→ beginnings of weak field perturbation theory ...Laplace ...Poincaré
→ d’Alembert and Clairaut never spoke again

(Source: Jean d’Alembert, T.L. Hankins, Oxford 1990)
• What to do?

• Look at QCD in a different space-time (Euclidean). Lattice simulations. (viz. Creutzfest, Sept. 4-5, 2014 BNL). By now, detailed hadron spectra, decay constants. Recent proposals for parton distributions. (Ji, Qiu, Ma 2013, 14)

• Look at scattering, right here in Minkowski space to use asymptotic freedom at high momentum transfer. The S-matrix, even at high energy: pretty hopeless in PT

\[ \langle B \text{ out}|A \text{ in}\rangle = f \left( \ln \left( \frac{Q}{\mu} \right), \ln \left( \frac{m}{\mu} \right), \alpha_s(\mu) \right) \]

\[ = f \left( 0, \ln \left( \frac{m}{Q} \right), \alpha_s(Q) \right) \]

\[ = f \left( \ln \left( \frac{Q}{m} \right), 0, \alpha_s(m) \right) \]

\[ m - \text{mass scales: } m_\pi, m_p, m_q, m_G = 0 \ldots \text{what to do with their logs is THE question for perturbation theory.} \text{ We shouldn't even try to compute anything (this way) that is } \propto \ln 0! \]
From leading-twist “vacuum” pQCD to power corrections (some examples)

How we use asymptotic freedom

- Infrared safety & asymptotic freedom:

\[ Q^2 \hat{\sigma}_{SD}(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2) \alpha_s^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right) \]

\[ = \sum_n c_n(1) \alpha_s^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right) \]

- \(e^+e^-\) total; jets: a sum over collinear rearrangements and soft emission organizes all long-time transitions, which must sum to \(\leq 1\) by unitarity.
• What we’re *really* looking at here (with local source $J$)

$$\sigma[f] = \lim_{R \to \infty} \int d^4x e^{-i q \cdot y} \int d\hat{n} f(\hat{n})$$

$$\times \langle 0 | J(0) T[\hat{n}_i T_{0i}(x_0, R\hat{n}) J(y)] | 0 \rangle$$

(Sveshnikov & Tkachov 95, Korchemsky, Oderda & GS 96, Bauer, Fleming, Lee & GS 08, Hofman & Maldacena 08)

With $T_{0i}$ the energy momentum tensor

• “Weight” $f(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k f / d\hat{n}^k$ bounded

• We have to ask only very “smooth” questions! (come back to this)
• Generalization: factorization

\[ Q^2 \sigma_{\text{phys}}(Q, m, f) = \omega_{\text{SD}}(Q/\mu, \alpha_s(\mu), f) \otimes f_{\text{LD}}(\mu, m) \]

\[ + \mathcal{O}\left(\frac{1}{Q^p}\right) \]

\( \mu = \) factorization scale;

\( m = \) IR scale (\( m \) may be perturbative)

• “New physics” in \( \omega_{\text{SD}}; f_{\text{LD}} \) “universal” – for a given target or observed particle

• Almost all collider applications. Enables us to compute

the Energy-transfer-dependence in \(|\langle Q, \text{out}|A + B, \text{in}\rangle|^2\).

• But again, requires a smooth weight for final states!
Resummation?

- Whenever there is factorization, there is evolution

\[ 0 = \mu \frac{d}{d\mu} \ln \sigma_{\text{phys}}(Q, m) \]

\[ \mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu} \]

- Wherever there is evolution there is resummation,

\[ \sigma_{\text{phys}}(Q, m) = \sigma_{\text{phys}}(q, m) \exp \left\{ \int_{q}^{Q} \frac{d\mu'}{\mu'} P(\alpha_s(\mu')) \right\} \]
• For example: “collinear” factorization for a (non-singlet) DIS structure function:

\[
F_2(x, Q^2) = \int_x^1 d\xi \ C_a \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu) \right) f_{a/A}(\xi, \mu)
\]

which factorizes into a simple product under moments,

\[
\tilde{F}_2(N, Q^2) = \int_0^1 dx x^{N-1} F_2(x, Q^2) = \tilde{C}_a \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu) \right) \tilde{f}_{a/A}(N, \mu)
\] (1)

• & then we know \( \tilde{P}(N, \alpha_s) = \gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots \),
and we get

\[
\tilde{F}_2(N, \mu) = \tilde{F}_2(N, \mu_0) \exp \left[ -\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \gamma(N, \alpha_s(\mu')) \right]
\]

• and with \( \alpha_s(\mu) = 4\pi/b_0 \ln(\mu^2/L_{\text{QCD}}^2) \), this is

\[
\tilde{F}_2(N, Q) = \tilde{F}_2 q/H(N, Q_0) \left( \frac{\ln(Q^2/L_{\text{QCD}}^2)}{\ln(Q_0^2/L_{\text{QCD}}^2)} \right)^{-2\gamma_N^{(1)}/b_0}
\]
- **It works quite well.** Approximate scaling at moderate $\alpha$, pronounced evolution for smaller $\alpha$:
With these methods can describe both particles and jets in pp at 200 GeV ... at 8 TeV

Especially for the single-particle inclusive cross sections at RHIC, the range of agreement was a surprise. A great impetus for polarization, AA, pA and eA studies. In ratios, at least we understand the denominator!
II. Looking outward toward TMDs, low-\(x\) and polarized scattering

- All the foregoing looks “inward” toward the hard scattering. The emphasis is the verification of QCD at the shortest distances and the prediction of new physics signals at high energy.

- The path to a fuller understanding of the theory involves looking away from the hard scattering . . .

- In two qualitative ways:
  1. Far back into the initial state, before the collision: the structure of nuclei in the language of partons.
  2. Far forward to the final state, after the collision: hadronization.

- The questions we seek to answer will still be phrased in the language of hadronic structure and formation in terms of partonic degrees of freedom.
• We can get an idea of what’s involved by considering corrections to collinear factorization

\[ Q^2 \sigma_{\text{phys}}(Q, m, f) = \omega_{\text{SD}}(Q/\mu, \alpha_s(\mu), f) \otimes f_{\text{LD}}(\mu, m) + O\left(\frac{1}{Q^p}\right) \]

• In general power corrections are quite complicated, but for DIS structure functions like \( F_2(x, Q^2) \) with \( Q^2 \gg \Lambda^2 \), they are a series of factorized expressions, corresponding to the scattering of multiple partons and in the transverse momentum carried by individual partons: \( k_T \) as well as \( x \).

• If partons are spread out over the surface of the “target”, multiple scatterings are unlikely, but if the density is large:

\[ \frac{G(x, Q_s^2)}{\pi R_{\text{target}}^2 Q_s^2} \sim 1 \]

multiple interactions are the rule: saturation.

• These “higher-twist” corrections should match onto the upturns in HERA structure function data . . . saturation, CGC. The “Initial state”.
• Higher twist and the final state: the nature of higher twist corrections is inverse powers not just of $Q^2$, but of $M^2_{\text{had}} \sim [(1 - x)Q^2]^2$, and so become more and more important for the “elastic limit” of $x \rightarrow 1$.

• Higher twist acts to ‘redistribute probability around a smooth extrapolation of leading twist. Bloom-Gilman duality.
And rescattering in nuclear matter:

Sum leading powers in the $A^{1/3}/Q^2$-dependence (Qiu, Vitev).
- And what about the requirement of “smooth” sum over states?

- Example: diffractive final states: gaps with no particles?

- For $Q^2 \ll W^2$ in DIS:

- Breaking the top rung of target ladder does not require radiation; combining the broken rung with the projectile requires radiation, but only in a small range of rapidity. (Gribov-Regge models of $A$-dependence). Evolution with $Q$ from the top even in presence of gap. The pomeron as a process, not a particle.
In the following, a sampling of generalizations of collinear factorization with examples of evolution.

Each parton distribution below has a fragmentation analog. We’ll return briefly to one difference below.

The main message is the unity of the methods.

And their accessibility through electron-proton / electron-nucleus scattering.
Transverse momentum factorization

- The classic extension of collinear factorization.
- For Drell-Yan and DIS, (see Collins (2011))

\[
\frac{d\sigma_{AB\to V}}{dQ^2d^2Q_T} = \frac{1}{S} \sigma_{ab\to V}^{(0)}(Q^2) \int dx_a \, dx_b \, h_{ab}^{(j)} \left( \frac{Q^2}{x_a x_b S}, \alpha_s(\mu) \right) \sum_{a,b} \times \int dx_a d^2k_{t,a} f_{a/A}(x_a, k_{t,a}, \mu) \int dx_b d^2k_{t,b} f_{b/B}(x_b, k_{t,b}, \mu) \times \delta^2(Q_T + k_{t,a} + k_{t,b} + k_{t,s}) + Y_j.
\]

- Interpretations and limitations
  - f’s: are now TMDs
  - h: short distance (off shell by order Q
  - Corrections nonsingular for $Q_T \to 0$
  - $Q_T$ is fixed already at the hard scattering.
  - In general does not extend to pairs of hadrons in pp – final state interactions don’t decouple.
  - This failure of “universality” is an opening to new physical phenomena, not a limitation. (Collins, Qiu, Rogers, Muelders …)
• The double sfactorization leads to evolution of double logs. For example (thanks to Ted Rogers) ... the cross section as an inverse transform:

\[
\frac{d\sigma}{dq_T^2} \sim \int d^2 b_T \ e^{-ib_T \cdot q_T} \times \\
\left\{ \begin{array}{l}
\times \int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{j/j}(x_1/\hat{x}_1, b_\ast; \mu_b^2, \mu_b, g(\mu_b)) f_{j/p}(\hat{x}_1, \mu_b) \\
\times \int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{j'/j'}(x_2/\hat{x}_2, b_\ast; \mu_b^2, \mu_b, g(\mu_b)) f_{j'/p}(\hat{x}_2, \mu_b) \\
\end{array} \right\} \\
\times \exp \left\{ \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[ B(g(\mu')) + \ln Q^2 \mu'^2 A(g(\mu')) \right] \right\} \times \\
\times \exp \left\{ -g_1(x_1, b_T) - g_2(x_2, b_T) - 2g_K(b_T) \ln \frac{Q}{Q_0} \right\} + Y \text{ term}
\]

(Collins, Soper, Sterman (CSS) formalism (1981-1985)... (many similar formalisms))

• The “many similar formalisms” include those based on soft-collinear effective theory, and with different large-distance regularizations. The similarities don’t rule out lively discussion. Much of this has to do with the important nonperturbative factors and their evolution. (In 2014: Aidala, Field, Gamberg, Rogers and Sun, Yuan, Yuan)
BFKL & high parton density

• “Multiperipheral” re-factorization for a DIS structure function.

\[ F(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} C\left(\frac{x}{\xi}, \mu^2/Q^2\right) G(\xi, Q^2) + \mathcal{O}\left(\frac{1}{Q^2}\right) \]
\[ G(\xi, Q^2) = \int^Q d^2k_T \psi(\xi, k_T) \]

\[ F(x, Q^2) = \int d^2k_T \ c\left(\xi, Q, k_T\right) \psi(\xi', k_T) + \mathcal{O}\left(\frac{1}{\ln^2 x}\right) \]
\[ \downarrow \]
\[ \xi \frac{d\psi(\xi, k_T)}{d\xi} = \int d^2k'_T \mathcal{K}(k_T - k'_T) \psi(\xi, k'_T) \]

• Roles of \( k_T \) and \( \xi \) exchanged – \( \xi' \) as factorization scale
• Equation, ansatz and solution: 
  \[ \tilde{\psi} \equiv (1/k_T^2)\psi \quad \tilde{\alpha}_s \equiv \alpha_s/\pi \]

  \[
  \xi \frac{d\tilde{\psi}(\xi, k_T)}{d\xi} = -\frac{\alpha_s N}{\pi^2} \int \frac{d^2k_T'}{(k_T - k_T')^2} \left[ \tilde{\psi}(\xi, k_T) - \frac{k_T'^2}{2k_T^2} \tilde{\psi}(\xi, k_T') \right] + \text{NLO}
  \]

  \[
  \tilde{\psi} \sim \xi^{-\omega} \left( \frac{k_T^2}{\mu^2} \right)^\gamma^{-1}
  \]

  \[
  \omega(\gamma) = \tilde{\alpha}_s \chi_0(\gamma) \left[ 1 - \beta_0 \tilde{\alpha}_s \ln \frac{k_T^2}{\mu^2} \right] + \tilde{\alpha}_s^2 \chi_1(\gamma)
  \]

  \[\downarrow\]

  \[\omega(\gamma) = \alpha_s \chi_0(\gamma) \left[ 1 - \ln \frac{k_T^2}{\mu^2} \right] + \alpha_s^2 \chi_1(\gamma)\]

• a fast growth for small \(\xi \leftrightarrow x\).

• Good to recall that beyond NLO must generalize the factorization & equation

• Nuclear targets enhance these effects through the build-up of low-\(x\) partons.
Effective theories for high parton density

- One thing that’s special about BFKL: evolve to low $x \leftrightarrow$ high parton density at “fixed” (actually diffusing) virtuality $xG(x) \sim x^{-\omega}$

- Theory of dense, weakly-interacting partons ($\alpha_s \ll 1$)

- LO BFKL from scattering of recoilless sources (Wilson lines):

  $W_+ - W_-$ scattering as an effective field theory (Balitsky 99)

  \[ W_{\pm}(x^\pm, x_t) = P \exp \left[ \int_{-\infty}^{\infty} dx^\pm A^\pm(x) \right] \]

- Extensions to dipole splitting: BK and JIMWLK equations.
• The nuclear connection: from small-\(x\) at HERA to initial states for nuclei

• Photon as a dipole (a classic theme) and saturated structure functions

\[
F_2 \sim Q^2 N_c \int_0^1 dz \int d^2 x_t \left( 1 - \exp \left[ -\alpha_s \frac{x_t^2}{\sigma_0} xG(x, 1/x_t^2) \right] \right)
\]

• Nucleus as a distribution of \(W_{\pm}\)'s (Venugopalan 99)

\[
S_{\text{nuclear field}} = S_{\text{QCD}} + \frac{i}{N_c} \int d^2 x_t dx^- \rho(x_t, x^-) W_+(x^-, x_t)
\]

• \(\Rightarrow\) classical strength gluon field \(<FF> \sim 2/\alpha_s\).
Polarized Distributions

• The first example: DIS cross section with spin

\[
\frac{d^2\sigma}{d\Omega dE} = \frac{\alpha_{\text{EM}}^2}{2mQ^4} \frac{E_e}{E_e'} L_{\mu\nu} W_{\mu\nu}
\]

\[
W_{\mu\nu} = W_{\mu\nu}^{\text{unpol}} + \frac{i}{E_e - E_e'} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma \, g_1(x, Q)
\]

\[
+ \frac{i}{(E_e - E_e')^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left[ p \cdot q s^\sigma - s \cdot q p^\sigma \right] g_2(x, Q),
\]

• Interpretation: contribution of quark helicity

\[
g_1(x, Q) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x, Q) + O(\alpha_s)
\]

• New measurements and (many) fits (including last night on hep-ph)

• Compass, HERMES, RHIC spin program have all provided qualitatively new insights, but there is far to go.

• There are many generalizations, for transverse and longitudinal polarizations, single and double and transverse momentum.
(Again, thanks to Ted Rogers)

**Taxonomy**

<table>
<thead>
<tr>
<th>Proton Quark</th>
<th>Unpolarized</th>
<th>Longitudinally polarized</th>
<th>Transversely polarized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unpolarized</td>
<td>$f_1(x, k_T)$</td>
<td>$\times$</td>
<td>$f_{1T}(x, k_T)$</td>
</tr>
<tr>
<td>Longitudinally polarized</td>
<td>$\times$</td>
<td>$g_{1L}(x, k_T)$</td>
<td>$g_{1T}(x, k_T)$</td>
</tr>
<tr>
<td>Transversely polarized</td>
<td>$h_{1L}(x, k_T)$</td>
<td>$h_{1L}(x, k_T)$</td>
<td>$h_{1T}(x, k_T)$</td>
</tr>
</tbody>
</table>

- Sivers and Boer-Mulders are nonvanishing because TP symmetry changes the nonabelian phase of partonic fields in general. (Collins)
Generalized Distributions

• The factorization of collinear distributions (and TMDs) depends on the localization of partons by hard scattering, and the absence of final-state interactions. Such factorizations are sensitive to local properties of hadrons and nuclei, not so much to their coherent structure. This is fine for parton polarizations; not so much for orbital angular momenta. These require generalized parton distributions. (Ji, Radyushkin)

• Comparing standard ("diagonal") and generalized (skewed) distributions:

1. (After using the optical theorem) the standard distribution in terms of creation/absorption operators looks like

\[ q(x) = \int d\ell \langle p | b_q^\dagger (xp + \ell) b_q(xp + \ell) |p \rangle \]

2. While the generalized parton distribution (GPD) is

\[ Q(x, x') = \int d\ell \langle (1 - \delta)p | b_q^\dagger ((x - \delta)p + \ell) b(xp + \ell) |p \rangle . \]

3. Quark collinear momentum fractions

\[ p_0 \int_0^1 x q(x) \]
• Generalized distributions were originally designed to isolate:

• **Quark and gluon total angular momenta**

\[
J_q = \int d^3x \left[ \frac{\Sigma}{2} \psi^\dagger \psi + \psi^\dagger x \times (-iD) \psi \right]
\]

\[
J_g = \int d^3x \ x \times (E \times B)
\]

• The off-diagonal connection \( (p' = p + \Delta) \)

\[
J_{q,g} = \frac{1}{2} \left[ A_{q,g}(0) + B_{q,g}(0) \right]
\]

\[
\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p + \Delta) \left[ A_{q,g}(\Delta^2) \gamma^\mu p'^\nu \
+ B_{q,g}(\Delta^2) p'^\mu \frac{i\sigma_{\mu\alpha}}{2M} \Delta^\alpha - (\mu \leftrightarrow \nu) \right] u(p)
\]

• Dependence on \( \Delta \) measures non-local correlations associated with angular momentum.

• To be extracted from DVCS (for example) and vector boson production.

• More generally, the full set of TMDs and GPDs offer the promise of a three dimensional picture of the nucleon.
III. A few words on factorization and universality

• The classical analog of factorization

\[ -\Delta = c\beta t' - x_3' \]

\[ \Delta \equiv x_3' - \beta ct' \]

• Why a classical picture isn’t far-fetched . . .

The correspondence principle is the key to IR divergences.

An accelerated charge must produce classical radiation, and an infinite numbers of soft gluons are required to make a classical field.
• **Transformation of a scalar field:**

\[
\phi(x) = \frac{q}{(x_T^2 + x_3^2)^{1/2}} = \phi'(x') = \frac{q}{(x_T^2 + \gamma^2 \Delta^2)^{1/2}}
\]

From the Lorentz transformation:

\[
x_3 = \gamma(\beta ct' - x'_3) \equiv -\gamma \Delta.
\]

Closest approach is at \( \Delta = 0 \), i.e. \( t' = \frac{1}{\beta c} x'_3 \).

The scalar field transforms “like a ruler”: At any fixed \( \Delta \neq 0 \), the field decreases like \( 1/\gamma = \sqrt{1 - \beta^2} \).

Why? Because when the source sees a distance \( x_3 \), the observer sees a much larger distance.
The “gluon” $\vec{A}$ is enhanced, yet is a total derivative:

$$A^\mu = q \frac{\partial}{\partial x'_\mu} \ln (\Delta(t', x'_3)) + \mathcal{O}(1 - \beta) \sim A^-$$

The “large” part of $A^\mu$ can be removed by a gauge transformation.
– The “force” $\vec{E}$ field of the incident particle does not overlap the “target” until the moment of the scattering.

– “Advanced” effects are corrections to the total derivative:

$$1 - \beta \sim \frac{1}{2} \left[ \sqrt{1 - \beta^2} \right]^2 \sim \frac{m^2}{2E^2}$$

– Power-suppressed. These are corrections to factorization.

– At the same time, a gauge transformation also induces a phase on charged fields:

$$q(x) \Rightarrow q(x) e^{i \ln(\Delta)}$$
– **Initial-state interactions decouple from hard scattering**

– **Summarized by multiplicative factors: the parton distributions.**

– The phase cancels for collinear pdf, but not necessarily for a TMD.

– But what about DIS cross sections where we observe a scattered particle in the final state? (SIDS)
– Much of the same reasoning holds for a scattering particle in the field of the initial state.

\[ x_3' < \beta c' t \]

– Subtle but important difference: \( \Delta \) changes sign in the final state.

– Then the gauge function in \( \ln(\Delta) \) gets an imaginary part.

– \( q(x) \Rightarrow q(x) e^{i \ln(\Delta)} \) no longer a pure phase.

– A change in phase between initial- and final-state interactions.

– This is an important limit to universality even when TMD factorization holds.
IV. In conclusion: tasks for an EIC

1. Open low-$x$ at high luminosity: evolution, shadowing and saturation; $F_L$.

2. Polarization and nucleon structure. Explore universality and, in concert with other facilities, its limitations.


4. Diffraction studies. Study the “$A - Q^2$ plane” to disentangle effects we know are there: $A^{4/3}$, $A(A - 1)$ . . .

5. Propagation of scattered partons in nuclear medium. Complement AA; complete fixed-target.

6. Quark-hadron duality studies. Follow the histories of scattered quanta; confinement in action.
• Why are we doing this? Why look outward from the hard scattering, and not only inward?

• JLab at 12, RHIC, Hermes and Compass began many of these programs, but the reach and luminosity of the planned EIC is documented.

• QCD is difficult, and sometimes we do have to measure before we compute. Or, in other words . . .

• QCD is the field theory “par excellence”. The nucleon and nucleus are libraries written in a language in which we know only a few phrases. An electron-proton/ion collider would provide an indispensable step towards a fuller translation of hadrons into the language of partons, and perhaps a new system of calculation as yet unimagined.