1. Blackbody radiation is emitted by all objects that have finite temperature. We will consider a number of items that you are familiar with and deduce the blackbody radiation they emit.

(a) YOU! Your temperature is 37 °C, or 310 K. Find the peak wavelength and frequency of light emitted by you. What range of the spectrum (radio, microwave, infrared, visible, ultraviolet, x-ray, gamma-ray) is this light found in?

The necessary equation is $\lambda_p T = 2.9 \times 10^{-3} mK$. So, for a human the peak wavelength is:

$$\lambda_p 310 = 2.9 \times 10^{-3}$$

$$\lambda_p = \frac{2.9 \times 10^{-3}}{310}$$

$$\lambda_p = 9.35 \times 10^{-6} m$$

This is microwave/infrared region.

(b) Your stove coil turns red hot. Take 600 nm as the peak wavelength of this light. What is the temperature of the stove coil?

$$\lambda_p T = 2.9 \times 10^{-3}$$

$$600 \times 10^{-9} T = 2.9 \times 10^{-3}$$

$$T = \frac{2.9 \times 10^{-3}}{600 \times 10^{-9}}$$

$$T = 4833K$$

(c) Penzias and Wilson discovered that radio static is actually blackbody radiation from the cold outreaches of space. They set the temperature of space at 2.7 K (don’t fret over this, space is not completely empty so it can have temperature). What is the peak frequency of the blackbody radiation from cold space?

$$\lambda_p T = 2.9 \times 10^{-3}$$

$$\lambda_p 2.7 = 2.9 \times 10^{-3}$$

$$\lambda_p = \frac{2.9 \times 10^{-3}}{2.7}$$

$$\lambda_p = 1.07 \times 10^{-3} m$$

$$\lambda_p f_p = c$$

$$f_p = \frac{c}{\lambda_p} = \frac{3.00 \times 10^8}{1.07 \times 10^{-3}}$$

$$f_p = 2.79 \times 10^{11} Hz$$
2. A Helium atom consists of its nucleus (\(\frac{2}{4}He\)) and 2 electrons. The energy levels in this nucleus are not precisely the same as though in the Bohr formula since the electrons are not only affected by the \(Z=2\) nucleus, but they are also affected by each other.

However, if one of the electrons from a He atom is removed, this positively charged ion (a \(\frac{2}{4}He\) nucleus plus only ONE electron), is indeed quite accurately described by the Bohr formula. We will calculate a number of things for this \(Z=2\) ion.

(a) If the electron is in the ground state, what energy photon is necessary to remove it?

The basic equation for this problem is \(E_n = -13.6eV \frac{Z^2}{\infty^2}\). To remove an electron from the ground state, you must make the transition from \(n = 1\) to \(n = \infty\).

\[
E_{\text{photon}} = E_\infty - E_1 \\
E_{\text{photon}} = -13.6eV \frac{Z^2}{\infty^2} - (-13.6eV \frac{Z^2}{1^2})
\]

\[
E_{\text{photon}} = 0 - (-13.6eV \frac{2^2}{1^2})
\]

\[
E_{\text{photon}} = 54.4eV
\]

(b) If the electron is in the \(n=2\) state, what energy photon is necessary to remove it?

\[
E_{\text{photon}} = E_\infty - E_2 \\
E_{\text{photon}} = -13.6eV \frac{Z^2}{\infty^2} - (-13.6eV \frac{Z^2}{2^2})
\]

\[
E_{\text{photon}} = 0 - (-13.6eV \frac{2^2}{2^2})
\]

\[
E_{\text{photon}} = 13.6eV
\]

(c) What energy photon is released if the electron drops from the \(n=3\) to the \(n=2\) state?

\[
E_{\text{photon}} = E_3 - E_2 \\
E_{\text{photon}} = -13.6eV \frac{Z^2}{3^2} - (-13.6eV \frac{Z^2}{2^2})
\]

\[
E_{\text{photon}} = -13.6eV \frac{2^2}{3^2} - (-13.6eV \frac{2^2}{2^2})
\]

\[
E_{\text{photon}} = -13.6eV \left(\frac{4}{9} - 1\right)
\]
\[ E_{\text{photon}} = 13.6eV \times \frac{5}{9} \]  
\[ E_{\text{photon}} = 7.55eV \] (27)  
(28)
3. Bohr derived his famous formula for quantized energy levels in an atom using the condition that angular momentum was quantized. deBroglie instead assumed that electron waves (of known wavelength) must fit into orbits in much the way that waves on a guitar fit within the string.

(a) Write down the Bohr condition on angular momentum.
Using \( \hbar = \frac{h}{2\pi} \) Bohr took as his premise that angular momentum was quantized into integer multiples of \( \hbar \). So his condition was:

\[
\begin{align*}
|\vec{L}| &= n\hbar \\
|\vec{r} \times \vec{P}| &= n\hbar \\
\text{mvr} &= n\hbar
\end{align*}
\]

(b) Write down the deBroglie condition on wavelength.
deBroglie instead assumed that the circumference of the orbit was an integer number of wavelengths:

\[2\pi r = n\lambda\]

(c) Show that these two conditions are indeed the same.
Using the quantum condition for momentum:

\[
\begin{align*}
p &= \frac{\hbar}{\lambda} \\
\lambda &= \frac{\hbar}{p} \\
2\pi r &= n\frac{\hbar}{p} \\
2\pi r &= n\frac{\hbar}{mv} \\
mvr &= n\frac{\hbar}{2\pi} \\
mvr &= n\hbar
\end{align*}
\]
4. No two electrons can occupy identically the same quantum state. For this reason, not all electrons on a given atom can be in the $n=1$ state, some of them must be in higher states. As one considers bigger and bigger atoms, the $n=1$ level will fill up, followed by $n=2$, followed by $n=3$, and so on...

(a) We will consider that atom that completely fills the $n=2$ level. Each electron in this atom will have a set of 4 quantum numbers $(n, l, m_l, m_s)$. Please fill in the table below specifying all four quantum numbers for each of these electrons:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$l$</th>
<th>$m_l$</th>
<th>$m_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>$-\frac{1}{2}$</td>
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<td>2</td>
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<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>+1</td>
<td>$-\frac{1}{2}$</td>
</tr>
</tbody>
</table>

(b) What element is this in the periodic table?
This is 10 electrons and so this is Neon.

(c) What special property results from having the $n=2$ level completely full?
Neon is a noble gas.
5. Some particular material has a work function of 5 eV. For this material:

(a) Calculate the lowest frequency photon that can remove an electron from this material.

When removing an electron the photon must at least supply an energy equal to the work function.

\[ E = hf \]  
\[ f = \frac{E}{h} \]

\[ h = 6.626068 \times 10^{-34} \text{Js} \] \[ \frac{1e}{1.602 \times 10^{-19} \text{C}} \]  
\[ h = 4.135667 \times 10^{-15} \text{eVs} \]

\[ f = \frac{5 \text{eV}}{4.135667 \times 10^{-15} \text{eVs}} \]  
\[ f = 1.21 \times 10^{-15} \text{eVs} \]

(b) Calculate the wavelength of a photon that removes electrons from the material such that the electrons leave with 8 eV as their maximum kinetic energy.

OK, since the work function is 5 eV, and the electrons leave with 8 eV, the photon must supply a total energy:

\[ E_{\text{photon}} = 5 \text{eV} + 8 \text{eV} = 13 \text{eV} \]

\[ E_{\text{photon}} = 13 \text{eV} = hf = \frac{hc}{\lambda} \]

\[ \frac{hc}{\lambda} = 13 \text{eV} \]

\[ \lambda = \frac{hc}{13 \text{eV}} \]

\[ \lambda = \frac{4.135667 \times 10^{-15} \times 3.0 \times 10^8}{13 \text{eV}} \]  
\[ \lambda = 9.53 \times 10^{-8} \text{m} = 95.3 \text{nm} \]
6. Shown in the figure below is a Compton scattering event in which a high energy photon is deflected by its interaction with an electron. The photon is deflected by the angle \( \phi \), and the electron (initially at rest) is ejected at the angle \( \theta \).

(a) Let the initial energy of the photon be 5 keV. What is the initial wavelength of the photon? What is the momentum of the initial photon?

Easy ones...:

\[
E = hf \tag{51}
\]

\[
h = 6.626068 \times 10^{-34}\text{Js} \frac{1e}{1.602 \times 10^{-19}\text{C}} \tag{52}
\]

\[
h = 4.135667 \times 10^{-15}\text{eVs} \tag{53}
\]

\[
15000\text{eV} = (4.135667 \times 10^{-15}\text{eVs}) \times f \tag{54}
\]

\[
f = 3.63 \times 10^{18}\text{Hz} \tag{55}
\]

\[
\lambda f = c \tag{57}
\]

\[
\lambda = \frac{c}{f} \tag{58}
\]

\[
\lambda = \frac{3.0 \times 10^8}{3.63 \times 10^{18}} \tag{59}
\]

\[
\lambda = 8.26 \times 10^{-11}\text{m} \tag{60}
\]

\[
p = \frac{h}{\lambda} \tag{61}
\]

\[
p = \frac{6.626068 \times 10^{-34}\text{Js}}{8.26 \times 10^{-11}\text{m}} \tag{62}
\]

\[
p = 8.021 \times 10^{-24}\text{kgm/s} \tag{63}
\]

(b) If the angle \( \phi \) is 30 degrees, calculate the wavelength of the scattered photon.

\[
\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \phi) \tag{64}
\]

\[
\lambda' = 8.26 \times 10^{-11}\text{m} + \frac{6.626068 \times 10^{-34}\text{Js}}{9.10938188 \times 10^{-31}\text{kg} \times 3.0 \times 10^8\text{m/s}} (1 - \cos 30^\circ) \tag{65}
\]
\[ \lambda' = 8.26 \times 10^{-11} m + 2.42 \times 10^{-12} (1 - 0.866) \]  
\[ \lambda' = 8.26 \times 10^{-11} m + 3.24 \times 10^{-13} m \]  
\[ \lambda' = 8.29 \times 10^{-11} m \] (68)

(c) Find the momentum vector (yes, x and y components) of the photon as it exits the collision zone.

\[ |p_{\text{photon}}^-| = \frac{h}{\lambda'} \] (69)

\[ |p_{\text{photon}}^-| = 7.99 \times 10^{-24} \text{kgm/s} \] (70)

\[ p_{\text{photon}}^- = |p_{\text{photon}}^-| \cos \phi \hat{i} - |p_{\text{photon}}^-| \sin \phi \hat{j} \] (71)

\[ p_{\text{photon}}^- = 7.99 \times 10^{-24} \cos 30^\circ \hat{i} - 7.99 \times 10^{-24} \sin 30^\circ \hat{j} \] (72)

\[ p_{\text{photon}}^- = \left( 6.92 \times 10^{-24} \hat{i} - 4.00 \times 10^{-24} \hat{j} \right) \text{kgm/s} \] (73)

(d) Find the momentum vector of the electron as it exits the collision zone.

Here we need to use conservation of momentum!! The previous step told us the momentum of the photon that exist the collision. We also know the momentum of the photon that enters the collision:

\[ p_{\text{init}}^- = \left( 8.021 \times 10^{-24} \hat{i} + 0 \hat{j} \right) \] (74)

\[ p_{\text{init}}^- = \left( 8.021 \times 10^{-24} \right) \text{kgm/s} \] (75)

We can therefore setup the conservation of momentum in both the X direction and in the Y direction. For the x direction:

\[ 8.021 \times 10^{-24} = 6.92 \times 10^{-24} + P_x \] (76)

\[ P_x = 1.101 \times 10^{-24} \text{kgm/s} \] (77)

For the y direction:

\[ 0 = -4.00 \times 10^{-24} + P_y \] (78)

\[ P_y = 4.00 \times 10^{-24} \text{kgm/s} \] (79)

So, we can write the full momentum vector of the electron as:

\[ \vec{P} = \left( 1.101 \times 10^{-24} \hat{i} + 4.00 \times 10^{-24} \hat{j} \right) \text{kgm/s} \] (80)
(e) What is the direction, $\theta$, of the electron?
   Well, this one is easy. All you need is the direction of the vector from
   the previous step:

   \[
   \tan \theta = \frac{P_y}{P_x} \\
   \tan \theta = \frac{4.00}{1.101} \\
   \theta = 74.6^\circ
   \]

(f) What is the energy of the electron?
   Here we use the relativistic energy-momentum formula:

   \[
   E^2 = P^2 c^2 + m^2 c^4 \\
   E^2 = (P_x^2 + P_y^2) c^2 + m^2 c^4 \\
   E^2 = (1.101^2 + 4.00^2) \times 10^{-48} c^2 + m^2 c^4 \\
   E^2 = 17.21 \times 10^{-48} c^2 + m^2 c^4 \\
   c = 3.00 \times 10^8 \\
   c^2 = 9.00 \times 10^{16} \\
   c^4 = 81.00 \times 10^{32} \\
   m = 9.10938188 \times 10^{-31} \\
   m^2 = 82.98 \times 10^{-62} \\
   E^2 = 17.21 \times 10^{-48} 9.00 \times 10^{16} + 82.98 \times 10^{-62}81.00 \times 10^{32} \\
   E^2 = 154.89 \times 10^{-32} + 6721.38 \times 10^{-30} \\
   E^2 = 154.89 \times 10^{-32} + 672138. \times 10^{-32} \\
   E^2 = 671983. \times 10^{-32} \\
   E = 819.746 \times 10^{-16} J
   \]

(g) What is the wavelength of the electron?
   Here we use the momentum of the electron:

   \[
   |\vec{P}| = \sqrt{P_x^2 + P_y^2} \\
   |\vec{P}| = 4.149 \times 10^{-24}
   \]

   Using the deBroglie condition:

   \[
   p = \frac{h}{\lambda}
   \]
\[
\lambda = \frac{2}{p}
\]

\[
\lambda = \frac{6.626068 \times 10^{-34}}{4.149 \times 10^{-24}}
\]

\[
\lambda = \frac{6.626068 \times 10^{-34}}{4.149 \times 10^{-24}}
\]

\[
\lambda = 1.597 \times 10^{-10} m
\]
7. Close study of some heavy nuclei showed that they were not stable and emitted “rays”. This was different from X-rays (that were purposely made by man) since these special elements voluntarily made radiation instead of by coercion. Different types of these rays were labelled with the greek letters \(\alpha, \beta, \text{and}\ \gamma\). We now know better what these rays are:

(a) Identify what each of the following are made of:
   i. alpha rays.
      Helium nuclei
   ii. beta rays.
      electrons
   iii. gamma rays.
      photons

(b) In free space, a neutron has a lifetime of only 10 minutes! Write down the equation that shows the decay of a neutron. Your equation should look like “\(n \rightarrow \text{something} + \text{something} + \text{something}\)”.  
   \[ n \rightarrow p + e^- + \bar{\nu}_e \]

(c) In a \(^{12}\text{C}\) nucleus, the neutrons cannot decay. Explain why.
   The neutron cannot decay because all available states for the produced proton are already filled.

(d) In a \(^{14}\text{C}\) nucleus, the “last” neutron can decay. Write an equation that describes this decay in the form “\(^{14}\text{C} \rightarrow \text{something} + \text{something} + \text{something}\)”.  
   \[ ^{14}\text{C} \rightarrow ^{14}\text{N} + e^- + \bar{\nu}_e \]

(e) Would the decay listed in the previous step be considered alpha, beta, or gamma decay?
   beta decay because the detected particle is the electron.

(f) A proton in free space does not decay. However, in the nucleus \(^{11}\text{C}\) it can indeed decay. Write an equation for this decay in the form “\(^{11}\text{C} \rightarrow \text{something} + \text{something} + \text{something}\)”.  
   \[ ^{11}\text{C} \rightarrow ^{11}\text{B} + e^+ + \nu_e \]

(g) Living material maintains the ratio \(\frac{^{14}\text{C}}{^{12}\text{C}} \sim 1.3 \times 10^{-12}\). \(^{14}\text{C}\) has a 5730 year half-life.
   i. Calculate the \(\frac{^{14}\text{C}}{^{12}\text{C}}\) ratio for a sample of material that is 2,000 years old.
      The formula that governs radioactive carbon dating is:

      \[
      \frac{^{14}\text{C}}{^{12}\text{C}} = R_0 e^{-\lambda t} \\
      \lambda = \frac{0.693}{t_{1/2}}
      \]

      \[
      \frac{^{14}\text{C}}{^{12}\text{C}} = R_0 e^{-\lambda t} \\
      \lambda = \frac{0.693}{t_{1/2}}
      \]

      \[
      \frac{^{14}\text{C}}{^{12}\text{C}} = R_0 e^{-\lambda t} \\
      \lambda = \frac{0.693}{t_{1/2}}
      \]

      \[
      \frac{^{14}\text{C}}{^{12}\text{C}} = R_0 e^{-\lambda t} \\
      \lambda = \frac{0.693}{t_{1/2}}
      \]
\[
\lambda = \frac{0.693}{5730 \text{years}} = 1.21 \times 10^{-4} \frac{1}{\text{years}}
\]  
(108)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} t}
\]  
(109)

In this case, we can calculate the remaining carbon using 2000 for \( t \):

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} t}
\]  
(110)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} \times 2000}
\]  
(111)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-0.242}
\]  
(112)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} \times 0.785
\]  
(113)

\[
\frac{^{14}C}{^{12}C} = 1.02 \times 10^{-12}
\]  
(114)

ii. Calculate the \( \frac{^{14}C}{^{12}C} \) ratio for a sample of material that is 20,000 years old.

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} t}
\]  
(115)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} \times 20000}
\]  
(116)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-2.42}
\]  
(117)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} \times 0.08892
\]  
(118)

\[
\frac{^{14}C}{^{12}C} = 0.1156 \times 10^{-12}
\]  
(119)

iii. Calculate the \( \frac{^{14}C}{^{12}C} \) ratio for a sample of material that is 65 Million years old.

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} t}
\]  
(120)

\[
\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12} e^{-1.21 \times 10^{-4} \times 65My}
\]  
(121)
\[ \frac{14}{6}C \quad = \quad 1.3 \times 10^{-12}e^{-7865} \] 
(122)

\[ \frac{14}{6}C \quad = \quad \text{ZERO (calculator fails)} \] 
(123)

iv. Explain why Carbon dating is **not** used for dinosaur bones.
There is too little of the original $^{14}C$ left to be measured.

(h) \( ^{229}_{96}Th \) (Thorium) decays by an alpha decay. Write an equation for this decay in the form \( ^{229}_{96}Th \rightarrow \text{something + something + something} \). The following information might be helpful:

\( \text{(86Rn, 87Fr, 88Ra, 89Ac, 90Th, 91Pa, 92U, 93Np)} \).

\( ^{229}_{96}Th \rightarrow \frac{4}{2}Th + \frac{225}{88}Ra \)
8. Early in the 20th century, much was known about chemistry and the periodic table (an outgrowth Mendeleev’s Table), but less was known about the interior structure of an individual atom. Rutherford’s famous experiment (performed by his assistants Geiger and Marsden) was the first look into the atom in detail and the results were surprising. Answering the questions below will teach you about how this historic event leads us to a better picture of the atom and more precisely, the nucleus.

(a) Using Avagadro’s number \( \left( 6 \times 10^{23} \right) \) and the density of Gold \( \left( 19.3 \frac{grams}{cm^3} \right) \), find the volume of one atom of gold \( \left( \frac{1}{197} Au \right) \).

Let’s take one mole of material = 197 grams.

\[
m = 197 \text{ grams} \tag{124}
\]

\[
\rho = 19.3 \frac{grams}{cm^3} = \frac{m}{V} \tag{125}
\]

\[
V = \frac{m}{\rho} = \frac{197}{19.3} \tag{126}
\]

\[
V = 10.2 cm^3 \tag{127}
\]

This can be divided up among \( N_A \) atoms and so:

\[
V_1 = \frac{10.2 cm^3}{6 \times 10^{23}} \tag{128}
\]

\[
V_1 = 1.7 \times 10^{-23} cm^3 \tag{129}
\]

(b) Assuming a Gold atom is a sphere, calculate the radius of this Gold atom using the result from the previous step.

\[
V_1 = \frac{4}{3} \pi r^3 \tag{130}
\]

\[
r = \left( \frac{3V_1}{4\pi} \right)^\frac{1}{3} \tag{131}
\]

\[
r = \left( \frac{3V_1}{4\pi} \right)^\frac{1}{3} \tag{132}
\]

\[
r = 1.60 \times 10^{-8} cm \tag{133}
\]

\[
r = 1.60 \times 10^{-10} m \tag{134}
\]

(c) Describe in 2-3 sentences the atom according to the Thomson “Plum Pudding” model (now known to be an incorrect description). Use the volume/radius from the previous steps somewhere in your reply. Thomson knew about the electrons as individual particles carrying the negative charge of an atom. He did not know what form the positive charge took. So, he made an assumption that the volume of the whole atom was filled with a positively charged goo.
(d) Shown in the figure below is a Thomson model atom and three possible incoming trajectories of an alpha particle (Helium nucleus). Which of these trajectories experiences the greatest deflection upon passing the Au atom?

Trajectory A feels little force. Trajectory C feels equal up and down forces (net force of zero). So, trajectory B has the greatest deflection.

(e) Calculations of expected deflections of the alpha particle from the Thomson model showed that the maximum expected deflection was 0.02°. Imagine the surprise when the data showed scatterings of more that 90°!! In the Thomson model of the atom one could not generate a force that was strong enough to make such a deflection. Explain in 3-4 sentences how the Thomson model was changed (by Rutherford) so that a single atom could generate larger forces ion the alpha particle.

The electric force between two point particles varies as $F = \frac{kQ_1Q_2}{r^2}$. Clearly one could not change $Q_1$ or $Q_2$. So the hope was in $r$. If the nucleus is large, you can have small distance to some but not all the charge. If the nucleus is small, $R$ can be small for every charge in the nucleus and therefore the nucleus was changed in the Rutherford model to be a perfect point.

(f) In Rutherford’s, famous formula, he assumed that the nucleus had zero size (a perfect mathematical point). Explain in 2-3 sentences why his formula failed when using higher energy alpha particles.

The nucleus is not a perfect point. The higher the energy of the alpha particle, the closer it comes to the center of the nucleus. Eventually at high enough energy, the alpha touches the surface of the nucleus and the Rutherford formula fails.

(g) Calculate all the following:

i. The radius of a Gold ($^{197}_{79}$Au) nucleus.

$$r = r_0 A^{\frac{2}{3}}$$
$$r = 1.2 \text{ fm} 197^{\frac{2}{3}}$$
$$r = 6.98 \text{ fm}$$

ii. The radius of an Alpha ($^{4}_2$He) particle.

This formula is poor for the He nucleus, but still helps to make the point:

$$r = r_0 A^{\frac{1}{3}}$$
$$r = 1.2 \text{ fm} 4^{\frac{1}{3}}$$
$$r = 1.9 \text{ fm}$$
iii. Calculate the energy of the alpha particle necessary to bring it so close to a Au nucleus that the edges touch (i.e. distance between nuclei = $R_{Au} + R_{He}$).

$$U = k \frac{Q_1 Q_2}{r}$$ (141)

$$Q_1 = 2 \times 1.602 \times 10^{-19} = 3.204 \times 10^{-19} C$$ (142)

$$Q_1 = 79 \times 1.602 \times 10^{-19} = 126.558 \times 10^{-19} C$$ (143)

$$U = \frac{9 \times 10^9 \frac{3.204 \times 126.558 \times 10^{-38}}{(1.9 + 7.0) \times 10^{-15}}}{(1.9 + 7.0) \times 10^{-15}}$$ (144)

$$U = 4.1 \times 10^{-12} J$$ (145)

$$U = 4.1 \times 10^{-12} J \frac{1eV}{1.602 \times 10^{-19} J}$$ (146)

$$U = 25.6 \times 10^6 eV = 25.6 MeV$$ (147)

(h) Explain what happens to the alpha particle if it comes closer to the Au nucleus than the result calculated in the previous step.

The alpha particle will feel the nuclear force and be absorbed by the nucleus.

(i) What new force was introduced into physics to explain the fate of the high energy alpha particle.

The nuclear force.
9. Now that we know about the nuclear force we can answer all of the following:

(a) Consider two nuclei. The electric force repels these two nuclei. The nuclear force attracts them. Since the nuclear force is much stronger why do these two not get attracted and immediately fuse? The nuclear force has finite range. Unless the energies are high, the coulomb force keeps nuclei too far separated to experience the attractive forces.

(b) OK, now we know why nuclei on a tabletop do not fuse (i.e. cold fusion). Now consider two hydrogen nuclei (i.e. two protons) each of which has a charge of $q = 1.602 \times 10^{-19} \text{ C}$. What would be the electric potential energy if these were separated by a distance of $r = 1 \text{ fm} = 1 \times 10^{-15} \text{ m}$.

$$ U = k \frac{Q_1 Q_2}{r} $$

$$ Q_1 = Q_2 = 1.602 \times 10^{-19} $$

$$ U = 9 \times 10^9 \frac{2.566 \times 10^{-38}}{1.0 \times 10^{-15}} $$

$$ U = 2.31 \times 10^{-13} \text{ J} $$

(c) The separation distance in the previous step of this problem is small enough to experience the nuclear force. Convert this amount of energy into a temperature using the Boltzmann constant ($k = 1.38 \times 10^{-23} \frac{\text{ J}}{\text{ K}}$). Careful, translational motion has three degrees of freedom. Thus:

$$ U = \frac{3}{2} k T $$

$$ T = \frac{2 U}{3 k} $$

$$ T = \frac{2}{3} \frac{2.31 \times 10^{-13}}{1.3806503 \times 10^{-23}} $$

$$ T = 11.1 \times 10^9 \text{ K} $$

You might be surprised at how high this temperature is. Well, even in the sun, the “typical” p-p collision is not capable of fusing. If it were, the sun would not last very long! It is, in fact, the rare collision in which the p-p have higher than typical energies (remember the Maxwell distribution of velocities??) that is capable of fusion.

(d) The temperature on the surface of the sun is 6000 K. The temperature in the center of the sun is about 15,000,000 K. Explain in 2-3
sentences why the sun has nuclear fusion reactions and your stove does not.

Your stove does not have enough energies, even accounting for maxwell distributions, to overcome coulomb and make fusion reactions.

(e) The simplest reaction in the sub involves fusing two protons. However, no nucleus exists with two protons and no neutrons. Write the reaction that actually occurs and provides the energy of the sun in the form “p + p → something + something + something”

\[ p + p \rightarrow ^{2}\text{H} + e^{-} + \nu_{e} \]

(f) In the previous equation, only one of the reaction products can reach us here on earth. Which one?

The neutrino reaches us since it does not feel electric or nuclear forces.
10. Inside a nucleus, the nuclear force attracts (strongly) but the electric force repels (weakly). The nuclear binding energy curve is shown in the figure below:

(a) Explain, using the characteristics of the nuclear and electric forces why this curve rises then falls.
The curve rises because the nuclear attraction makes larger nuclei more tightly bound. However, eventually the unlimited range electric force wins over the finite range nuclear force and nuclei become less bound. Above iron nuclei, the binding energy curve falls.

(b) Explain, using the characteristics of the nuclear and electric forces, why the rise is steeper than the fall.
This rise is driven by the strong nuclear force. The fall is driven by the weaker electric force. This is why the rise is steep and the fall is not.

(c) Which is the most tightly nucleus?
iron

(d) To release energy from a Uranium nucleus, would you use fission or fusion? Explain using the nuclear binding energy curve.
Uranium nuclei are too big. Thus fission releases energy.

(e) To release energy from two deuterons (\( ^1_2H \)), would you use fission or fusion? Explain using the nuclear binding energy curve.
Fusion.

(f) Which of the previous two reactions releases the most energy? Explain.
The fusion reaction releases more energy.

(g) One atomic mass unit is \( u = 1.6605402 \times 10^{-27} \) kg. The rest mass of a deuteron \( ^1_2H \) is 2.0141018 u and the rest mass of a helium \( ^4_2He \) nucleus is 4.026032 u. Calculate the energy released in the reaction \( ^1_2H + ^1_2H \rightarrow ^4_2He \).
Two deuterons account for a mass more than the Helium

\[
M_{\text{deut}} = 2 \times 2.0141018u = 4.0282036u \\
M_{\text{He}} = 4.026032u \\
M_{\text{deut}} - M_{\text{He}} = 0.0021716u \\
1u = 931.5MeV \\
E = 0.0021716u \times 931.5MeV = 2.022MeV
\]

(h) The result from the previous step is the energy released by a single nuclear reaction. What is the energy released is 2 moles of \( ^1_2H \) were used (i.e. producing 1 mole of \( ^4_2He \)).
\begin{align*}
E_{tot} &= E_1 \times N_A = 2.022\text{MeV} \times 6 \times 10^{23} \quad (161) \\
E_{tot} &= 1.21 \times 10^{24}\text{MeV} = 1.21 \times 10^{30}\text{MeV} \quad (162) \\
\frac{E_{tot}}{1e} &= 1.21 \times 10^{30}\text{eV} \quad (163) \\
E_{tot} &= 1.94 \times 10^{11}\text{J} \quad (164)
\end{align*}

WOW!

(i) What is the environmental impact of He?

Nothing
11. Nuclear reactors and bombs operate on the principle of a chain reaction induced by nuclear fission.

(a) Explain the process of nuclear fission:
   i. Which nuclei will spontaneously fission, large ones or small ones?
      Large nuclei spontaneously fission driven apart by the electric force.
   ii. Compare the reaction products of fission with alpha, beta, or gamma, decay.
      Fission is unusual in that the reaction products include large nuclear fragments.

(b) $^{235}\text{U}$ produces 2-3 neutrons when it fissions in addition to two large nuclear fragments. Why are these neutrons important for a reactor or bomb?
   These neutrons are able to induce the fission of another nucleus and thereby are the basis of the chain reaction.

(c) Explain the following terms:
   i. Critical mass.
      The number of decays per second is constant.
   ii. Sub-critical mass.
      The number of decays per second falls.
   iii. Super-critical mass.
      The number of decays per second rises.

(d) You have just learned that this morning your nearby nuclear power plant was at supercritical mass for a short time. Is this normal? Explain.
   To start the reactor, you must take it supercritical (so that the neutron flux increases). After the neutron flux reaches the desired level, you pull back to critical mass.

(e) What was the mechanism in the “Little Boy” bomb by which the Uranium was brought rapidly from Sub-critical to Super-critical?
   It was a gun-type bomb with a $^{235}\text{U}$ fired into a $^{235}\text{U}$ target.

(f) What trick is used in a power plant reactor so that it can run with less highly purified Uranium (i.e. not as purely $^{235}\text{U}$ but actually having mostly $^{238}\text{U}$, much closer to the natural abundance?)
   The neutrons are moderated (lowered in energy) to increase their cross section for subsequent reactions.

(g) A breeder reactor uses the $^{238}\text{U}$ in it’s core to “breed” additional fuel and thereby keep the reactor running for more years before the spent rods need to be replaced. Write the equation for the nuclear reaction the produces more fuel in the form
   $^{\text{“}}_{238}\text{U} + \text{something} \rightarrow \text{something} + \text{something}$. 

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This involves 2 beta decays after the Uranium absorbs a neutron:

\[
\begin{align*}
^{238}_{92}U + n & \rightarrow ^{239}_{92}U \quad (165) \\
^{239}_{92}U & \rightarrow ^{239}_{93}Np + e^- + \bar{\nu}_e \quad (166) \\
^{239}_{93}Np & \rightarrow ^{239}_{94}Pu + e^- + \bar{\nu}_e \quad (167)
\end{align*}
\]
12. The “rule of thumb” for an NMR device is that protons will achieve resonance at a frequency of 42.58 MHz when they are immersed in a magnetic field of 1.0 Tesla. Assume that a patient is placed in a magnetic field, B, whose strength varies with position, x, according to the formula 
\[ B(x) = 1.0 \text{ Tesla} + 0.1 \frac{\text{Tesla}}{m}x. \]

(a) Especially strong resonant signals are received coming back from the patient for frequencies of 43.00 MHz and 45.00 MHz. Find the locations in x that produce these signals.

\[ hf_1 = kB_1 \]  
\[ hf_2 = kB_2 \]  
\[ \frac{f_1}{f_2} = \frac{B_1}{B_2} \]

\[ \frac{43.00 \text{ MHz}}{42.58 \text{ MHz}} = \frac{B_1}{1.0T} \]

\[ B_1 = 1.00985T \]

\[ 1.00986 = 1.0 + 0.1x_1 \]

\[ 0.00986 = 0.1x_1 \]

\[ x_1 = 0.0986m = 9.86cm \]

\[ \frac{45.00 \text{ MHz}}{42.58 \text{ MHz}} = \frac{B_1}{1.0T} \]

\[ B_1 = 1.05683T \]

\[ 1.05683 = 1.0 + 0.1x_1 \]

\[ 0.05683 = 0.1x_1 \]

\[ x_1 = 0.5683m = 56.83cm \]

(b) The doctor can think of this technique as one that “maps the location of water” inside the patient. Explain why.

The NMR signal is strongest from the proton. This is most often found in the body as one of the H molecules in \( H_2O \).

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13. PET scanning is an additional method of medical imaging that involves the annihilation of a positron inside a patient producing two photons. The mass of a positron is the same as the mass of an electron, \( m_e = 9.109 \times 10^{-31} \text{ kg} \).

(a) What is the mass energy of the positron?

\[
E = mc^2 \tag{181}
\]
\[
E = 9.109 \times 10^{-31} \text{ kg} \left(3.0 \times 10^8\right)^2 \tag{182}
\]
\[
E = 8.19 \times 10^{-14} \text{ J} \frac{1 \text{ e}}{1.602 \times 10^{-19} \text{ C}} \tag{183}
\]
\[
E = 511000 \text{ eV} = 511 \text{ keV} \tag{184}
\]

(b) How much energy is released during the annihilation process?

The mass energy of the electron AND the positron are released:

\[
E_{\text{tot}} = E_p = 2 \times 511 \text{ keV} = 1022 \text{ keV} \tag{185}
\]

(c) This energy is divided equally between the two photons. What is the frequency of each of these photons?

\[
E_{\text{photon}} = 511 \text{ keV} = hf \tag{186}
\]
\[
h = 6.626068 \times 10^{-34} \text{ J s} \frac{1 \text{ e}}{1.602 \times 10^{-19} \text{ C}} \tag{187}
\]
\[
h = 4.135667 \times 10^{-15} \text{ eV s} \tag{188}
\]
\[
f = \frac{E}{h} = \frac{511000}{4.135667 \times 10^{-15}} = 1.2 \times 10^{20} \text{ Hz} \tag{189}
\]

(d) Explain in 2-3 sentences why PET imaging is higher resolution than SPECT.

PET images are produced by tracing the paths of the two photons. SPECT uses only 1 photon and is thereby inherently lower resolution.
14. Shown in the picture below is an X-ray tube.

(a) Where do the electrons come from?
The hot filament is able to release electrons.

(b) Why are the electrons pushed to the right?
The filament is negative and the W/Cu target is positive. So, the electric force pushes them.

(c) What physical process causes the electron energy to turn into EM radiation?
Bremsstrahlung or “braking radiation”.

(d) Suppose the accelerating voltage of the X-ray tube were 70,000 Volts.
   i. What is the energy of each electron as it strikes the tungsten/copper plate?

\[ U = qV \quad (190) \]
\[ q = 1e \quad (191) \]
\[ U = 1e \times 70000V = 70000eV \quad (192) \]
\[ U = 70keV \quad (193) \]

ii. Assuming that all this energy goes into a single photon, what is the frequency of that photon?

\[ E_{photon} = hf \quad (194) \]
\[ h = 6.626068 \times 10^{-34} Js \frac{1e}{1.602 \times 10^{-19}C} \quad (195) \]
\[ h = 4.135667 \times 10^{-15} eVs \quad (196) \]
\[ f = \frac{E}{h} = \frac{70000}{4.135667 \times 10^{-15}} = 1.69 \times 10^{19} Hz \quad (197) \]

iii. What is the wavelength of that photon?

\[ \lambda f = c \quad (198) \]
\[ \lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{1.69 \times 10^{19}} \quad (199) \]
\[ \lambda = 1.77 \times 10^{-11} m \quad (200) \]
15. You and a friend are each holding meter sticks that are held so as to align with the “z” axis. You are also both holding clocks. Your friend boards a train travelling at 0.8c in the Z direction:

(a) You watch your clock and your friends. When your clock shows 5 minutes have elapsed, you look at your friend’s clock. How much time seems to have elapsed on that clock?

\[
\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}
\]

\[
\gamma = \frac{1}{\sqrt{1 - (0.8)^2}}
\]

\[
\gamma = 1.666
\]

\[
\Delta t = \frac{5\text{min}}{\gamma}
\]

\[
\Delta t = 3\text{min}
\]

(b) Your friend looks at his clock and waits until it says (to him) that 5 minutes have passed. He then looks at your clock. How much time does he claim has elapsed on your clock?

3 minutes.

(c) You look at his meter stick and measure it. How long does his meter stick appear to you?

\[
L = \frac{1\text{meter}}{\gamma}
\]

\[
L = 0.6m = 60\text{cm}
\]

(d) He looks at your meter stick. How long does your meter stick appear to him?

60 cm

(e) Your friend’s trip will take him a distance of 4 light-years to reach another star. When he reaches that star indeed, his clock reads as though it took too little time to get there. You claim that it is because his clock ran slowly. He, on the other hand, has a completely different reason why he seems to have arrived early. What is his reasoning that the trip was so short?

You say his clock runs slow. He says your universe shrunk making the distance smaller and the trip shorter.