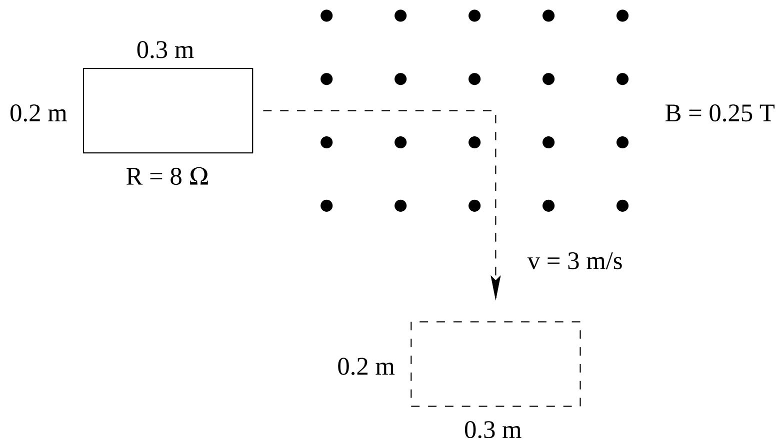
Physics 122

Midterm Examination #1

March 21, 2007

SOLUTIONS

1. Shown in the figure below is a rectangular loop located to the left of a region with a magnetic field ($B = 0.25\ Tesla$) pointed out of the paper. The loop is dragged with a constant speed of $v = 3\ \frac{m}{s}$ along the path shown by the dashed arrow. The resistance of the loop is $R = 8\ \Omega$.



(a) Determine the magnitude **AND** direction (clockwise or counter-clockwise) of the current in the loop during each of the following three conditions:

i. The loop is entering the field region.

$$\Phi_B = Blx \tag{1}$$

$$\frac{d\Phi_B}{dt} = Bl\frac{dx}{dt} = Blv \tag{2}$$

$$V = Blv \tag{3}$$

$$I = \frac{Blv}{R} = \frac{0.25 \cdot 0.2 \cdot 3}{8} = 0.01875\ A = 18.75\ mA \tag{4}$$

$$clockwise \tag{5}$$

ii. The loop is completely inside the field region.

$$I = 0 \tag{6}$$

iii. The loop is leaving the field region.

$$\Phi_B = Blx \tag{7}$$

$$\frac{d\Phi_B}{dt} = Bl\frac{dx}{dt} = Blv \tag{8}$$

$$V = Blv \tag{9}$$

$$I = \frac{Blv}{R} = \frac{0.25 \cdot 0.3 \cdot 3}{8} = 0.0281\ A = 28.1\ mA \tag{10}$$

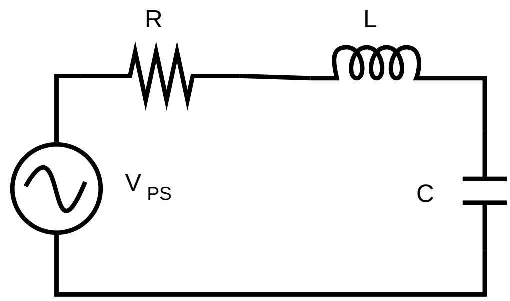
$$counter - clockwise \tag{11}$$

(b) Determine the magnitude **AND** direction of the force on the loop as it is exiting the field. Indicate the direction of the force by circling one of the following:

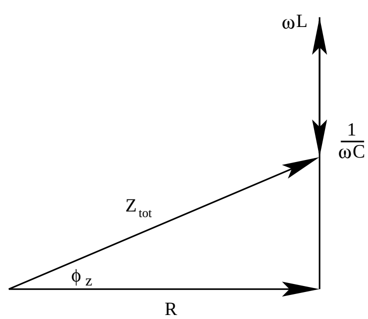
i. Toward the left of the page.
ii. **Toward the top of the page.**
iii. Toward the right of the page.
iv. Toward the bottom of the page.
v. Out of the page.
vi. Into the page.

$$F = IlB = 0.0281 \cdot 0.3 \cdot 0.25 = 2.1 \times 10^{-3}\ N \tag{12}$$

2. Shown below is an LRC circuit.



(a) Draw a phaser diagram representing this circuit.



(b) Analyze your phaser diagram to determine the magnitude of the impedance, $|\mathbf{Z}|$.

$$|Z_{tot}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{13}$$

(c) Analyze your phaser diagram to determine the phase of the impedance, ϕ_Z.

$$\phi_Z = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \tag{14}$$

(d) Let the resistance be $R = 600\ \Omega$, the inductance be $L = 1.04\ \mu H$ and the capacitance be $C = 3 \times 10^{-12}\ F$. Determine the peak voltage across the resistor if the circuit is driven by an AC power source with $V_{peak} = 10\ V$ and $f = 500\ MHz$.

$$|Z_{tot}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{15}$$

$$\omega L = 2\pi 500 \times 10^6 \left(1.04 \times 10^{-6}\right) = 3267\Omega \tag{16}$$

$$\frac{1}{\omega C} = \frac{1}{2\pi 500 \times 10^6 3 \times 10^{-12}} = 106\Omega \tag{17}$$

$$|Z_{tot}| = \sqrt{600^2 + (3267 - 106)^2} \tag{18}$$

$$|Z_{tot}| = 3217\Omega \tag{19}$$

$$V_R = \frac{R}{Z_{tot}} V_p = \frac{600}{3217} 10 = 1.87\ V \tag{20}$$

(e) Clearly, your circuit is not tuned for the $f = 500\ MHz$ signal. Determine the frequency that your circuit **IS** tuned for.

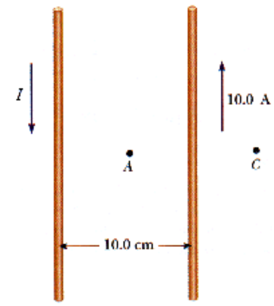
$$\omega = \frac{1}{\sqrt{LC}} \tag{21}$$

$$\omega = \frac{1}{\sqrt{1.04 \times 10^{-6} 3 \times 10^{-12}}} = 566 \times 10^6 \tag{22}$$

$$f = \frac{\omega}{2\pi} = 90.1 \times 10^6 Hz = 90.1\ MHz \tag{23}$$

(f) **EXTRA CREDIT (1 point):** What are the call letters of this radio station?
WUSB

3. Two parallel conductors carry currents in opposite directions, as shown in the Figure below. One conductor carries a current of 10.0 A. Point A is the midpoint between the wires, and point C is 5.00 cm to the right of the 10.0 A current. I is adjusted so that the magnetic field at C is zero.



(a) Find the value of the current I.

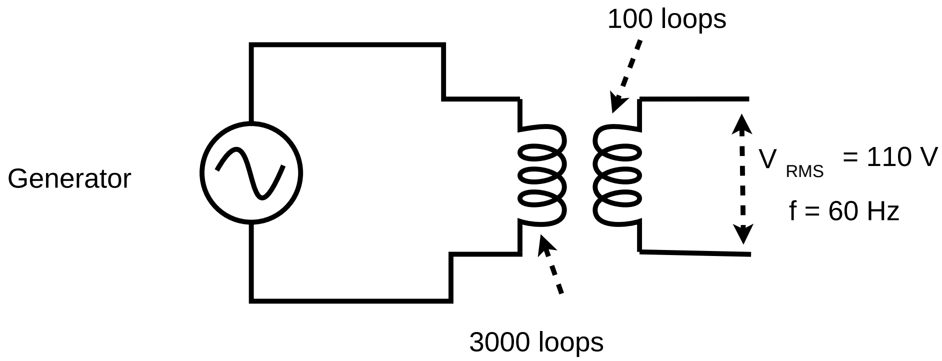
$$0 = \frac{\mu_0 I}{2\pi 0.15} - \frac{\mu_0 10}{2\pi 0.05} \tag{24}$$
$$\frac{\mu_0 I}{2\pi 0.15} = \frac{\mu_0 10}{2\pi 0.05} \tag{25}$$
$$\frac{I}{0.15} = \frac{10}{0.05} \tag{26}$$
$$I = 30\ Amps \tag{27}$$

(b) Find the value of the magnetic field at A.

$$B = \frac{\mu_0 30}{2\pi 0.05} + \frac{\mu_0 10}{2\pi 0.05} \tag{28}$$
$$B = \frac{\mu_0 40}{2\pi 0.05} \tag{29}$$
$$B = 0.00016\ Tesla \tag{30}$$

4. The circuit below shows a generator driving a transformer. The output side of the transformer has an AC voltage whose RMS is $V_{RMS} = 110\ V$ and whose frequency is $f = 60\ Hz$.

   The generator is made from a circular coil of wire with $N = 400\ turns$ whose radius is $r = 0.2\ m$. The coil is rotated in a uniform magnetic field, B.



   (a) Determine the *peak* voltage applied to the input side of the transformer.

$$V_{in}(RMS) = \frac{3000}{100}110 = 3300 \tag{31}$$

$$V_{in}(peak) = \sqrt{2}V_{in}(RMS) = 4667\ Volts \tag{32}$$

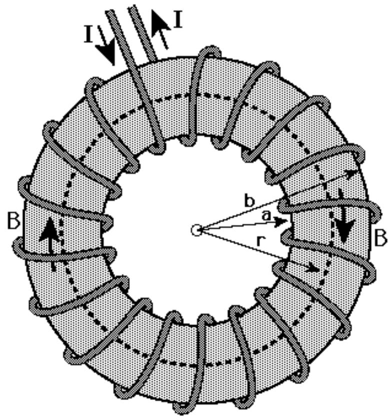
   (b) Determine the magnetic field in the generator.

$$V_{peak} = \omega NBA \tag{33}$$

$$B = \frac{V_{peak}}{\omega NA} = \frac{V_{peak}}{2\pi fN\pi r^2} \tag{34}$$

$$B = \frac{4667}{2\pi 60(400)\pi 0.2^2} = 0.246\ Tesla \tag{35}$$

5. Shown in the figure below is a toroid. The toroid carries a current I and has a total of N turns. Use Ampere's Law to calculate the magnetic field at the radius r shown in the Figure.



$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I_{in} \tag{36}$$

$$BL = \mu_0 I_{in} \tag{37}$$

$$L = 2\pi r \tag{38}$$

$$I_{in} = NI \tag{39}$$

$$B = \frac{\mu_0 NI}{2\pi r} \tag{40}$$

HINT: Think of a toroid as a long solenoid bent into a circular shape.