Physics 122
Midterm Examination #1
February 22, 2007

SOLUTIONS
1. Shown in the figure below are two point charges. The locations of the charges are specified on the graph in units of meters. Answer all the following:

(a) Determine the magnitude and direction of the total electric field at the point P.

\[ |\vec{E}_1| = k \frac{6 \mu C}{10^2} = 540 \frac{V}{m} \]  \hspace{1cm} (1)

\[ r_2 = \sqrt{5^2 + 7^2} = \sqrt{74} \]  \hspace{1cm} (2)

\[ |\vec{E}_2| = k \frac{3 \mu C}{74} = 365 \frac{V}{m} \]  \hspace{1cm} (3)

\[ \cos \theta_2 = \frac{7}{\sqrt{74}} \]  \hspace{1cm} (4)

\[ \sin \theta_2 = \frac{5}{\sqrt{74}} \]  \hspace{1cm} (5)

<table>
<thead>
<tr>
<th>( E_1 )</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>540</td>
<td></td>
</tr>
<tr>
<td>( E_2 )</td>
<td>(-365 \frac{5}{\sqrt{74}} = -212)</td>
<td>(-365 \frac{7}{\sqrt{74}} = -297)</td>
</tr>
<tr>
<td>( E_{\text{tot}} )</td>
<td>(-212)</td>
<td>243</td>
</tr>
</tbody>
</table>

\[ |\vec{E}_{\text{tot}}| = \sqrt{212^2 + 243^2} = 322.5 \frac{V}{m} \]  \hspace{1cm} (6)

\[ \theta_{\text{tot}} = \tan^{-1} \frac{243}{-212} + 180 = 131^\circ \]  \hspace{1cm} (7)
(b) Determine the potential at the point P.

\[ V = k \frac{6\mu C}{10} + k \frac{-3\mu C}{\sqrt{74}} \]  
\[ V = 5400 - 3139 = 2261V \]
2. Shown in the figure below is an insulating hollow sphere with inner radius \( a \) and outer radius \( b \). The region \( a < r < b \) is uniformly filled with a total charge \( +Q \). Use Gauss Law to find the electric field in each of these regions:

(a) Region I \( (r < a) \).

\[
E = 0 \quad (10)
\]

(b) Region II \( (a < r < b) \).

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad (11)
\]

\[
EA = \frac{Q_{\text{in}}}{\epsilon_0} \quad (12)
\]

\[
A = 4\pi r^2 \quad (13)
\]

\[
\frac{Q_{\text{in}}}{Q_{\text{tot}}} = \frac{V_{\text{taken}}}{V_{\text{tot}}} \quad (14)
\]

\[
V_{\text{taken}} = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \quad (15)
\]

\[
V_{\text{tot}} = \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \quad (16)
\]

\[
Q_{\text{in}} = Q \frac{r^3 - a^3}{b^3 - a^3} \quad (17)
\]

\[
E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \left( b^3 - a^3 \right) \quad (18)
\]

(c) Region III \( (r > b) \).

\[
E = k \frac{Q}{r^2} \quad (19)
\]
NOTE: You must show *explicit work* in part (b). However parts (a) and (c) can receive full credit for the correct solution calculated in your head and simply presented on the paper.
3. Shown in the figure below is a capacitor network. The voltage on the battery is $V=10 \text{ V}$.

(a) Use the rules for capacitors in series and parallel to fill in the values to the following table:

<table>
<thead>
<tr>
<th>Element</th>
<th>C ($\mu$Farads)</th>
<th>V (Volts)</th>
<th>Q ($\mu$Coulombs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>6 $\mu$F</td>
<td>5</td>
<td>30 $\mu$C</td>
</tr>
<tr>
<td>$C_2$</td>
<td>2 $\mu$F</td>
<td>5</td>
<td>10 $\mu$C</td>
</tr>
<tr>
<td>$C_3$</td>
<td>4 $\mu$F</td>
<td>5</td>
<td>20 $\mu$C</td>
</tr>
<tr>
<td>$C_4$</td>
<td>7 $\mu$F</td>
<td>10</td>
<td>70 $\mu$C</td>
</tr>
<tr>
<td>$C_A$</td>
<td>6 $\mu$F</td>
<td>5</td>
<td>30 $\mu$C</td>
</tr>
<tr>
<td>$C_B$</td>
<td>3 $\mu$F</td>
<td>10</td>
<td>30 $\mu$C</td>
</tr>
<tr>
<td>$C_C$</td>
<td>10 $\mu$F</td>
<td>10</td>
<td>100 $\mu$C</td>
</tr>
</tbody>
</table>

The **Rules**: You do not need to show explicit calculations used to get the results for this problem (many of the calculations can be done in your head). This means that the scoring for this problem is based simply upon whether the numerical results are right or wrong (full credit for each correct number, zero credit for each incorrect number). **BE CAREFUL** in all your calculations.
4. A hot-water heater is rated at 4620 W and operates at 240 V.

(a) Find the resistance of the heating element, and the current.

\[ P = \frac{V^2}{R} \]  (20)
\[ R = \frac{V^2}{P} = \frac{240^2}{4620} = 12.47\Omega \]  (21)
\[ V = IR \]  (22)
\[ I = \frac{V}{R} = \frac{240}{12.47} = 19.25\,\text{amps} \]  (23)

(b) How long does it take to heat 125 kg of water from 12°C to 50°C, neglecting conduction and other losses?

\[ Q = mc\Delta T = 125 \cdot 4186 \cdot (50 - 12) = 19.88 \times 10^6\,\text{J} \]  (24)
\[ P = 4620\,\text{W} = \frac{4620\,\text{J}}{1\,\text{sec}} \]  (25)
\[ \frac{Q}{P} = 19.88 \times 10^6\,\text{J} \cdot \frac{1\,\text{sec}}{4620\,\text{J}} = 4303\,\text{sec} = 1.2\,\text{hours} \]  (26)

**NOTE:** You may or may not find the following constants useful:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{ice})</td>
<td>(2050,\text{J}/\text{kg}\cdot\text{K})</td>
</tr>
<tr>
<td>(c_{water})</td>
<td>(4186,\text{J}/\text{kg}\cdot\text{K})</td>
</tr>
<tr>
<td>(c_{steam})</td>
<td>(2080,\text{J}/\text{kg}\cdot\text{K})</td>
</tr>
<tr>
<td>(L_{fusion})</td>
<td>(334000,\text{J}/\text{kg})</td>
</tr>
<tr>
<td>(L_{vaporization})</td>
<td>(2258000,\text{J}/\text{kg})</td>
</tr>
<tr>
<td>(T_{mel})</td>
<td>0°C</td>
</tr>
<tr>
<td>(T_{boil})</td>
<td>10°C</td>
</tr>
</tbody>
</table>
5. Shown in the figure below is a circuit diagram. The resistors have values: 
\( R_1 = 5 \Omega, \ R_2 = 4 \Omega, \ R_3 = 3 \Omega \) and carry currents \( I_1, I_2, \) and \( I_3 \) respectively. The batteries have values: \( V_1 = 5 \ V, \ V_2 = 10 \ V. \)

(a) Place neatly drawn arrows on the circuit diagram above that clearly indicate your choices for the directions of the three currents in the circuit.

(b) Determine the currents \( I_1 \) (through \( R_1 \)), \( I_2 \) (through \( R_2 \)), and \( I_3 \) (through \( R_3 \)).

\[
I_1 + I + 2 = I_3 \quad (27)
\]
\[
5 - 5I_1 + 4I_2 - 10 = 0 \quad (28)
\]
\[
10 - 4I_2 - 3I_3 = 0 \quad (29)
\]

\[
I_1 + I_2 - I_3 = 0 \quad (30)
\]
\[
-5I_1 + 4I_2 + 0I_3 = 5 \quad (31)
\]
\[
0I_1 + 4I_2 + 3I_3 = 10 \quad (32)
\]
\[ I_1 = \begin{bmatrix} 0 & 1 & -1 \\ 5 & 4 & 0 \\ 10 & 4 & 3 \\ -5 & 4 & 0 \\ 0 & 4 & 3 \end{bmatrix} = \frac{5}{17} = 0.106 \text{ Amps} \]

\[ I_2 = \begin{bmatrix} 1 & 0 & -1 \\ -5 & 5 & 0 \\ 0 & 10 & 3 \\ -5 & 4 & 0 \\ 0 & 4 & 3 \end{bmatrix} = \frac{65}{47} = 1.383 \text{ Amps} \]

\[ I_3 = \begin{bmatrix} 1 & 1 & 0 \\ -5 & 4 & 5 \\ 0 & 4 & 10 \\ -5 & 4 & 0 \\ 0 & 4 & 3 \end{bmatrix} = \frac{76}{47} = 1.489 \text{ Amps} \]