1. Shown in the figure below are two point charges located along the x axis. Determine all the following:

(a) Find the magnitude and direction of the electric field at the origin. This problem reduces to one dimension since the charges themselves and the point at which we make our calculation are all along a single line. Electric field points away from positive charges and toward negative charges. For both of our charges this will mean an electric field to the right, or in the +î direction.

\[
\vec{E} = \left( k \frac{|Q_1|}{r_1^2} + k \frac{|Q_2|}{r_2^2} \right) \hat{i} \quad (1)
\]

\[
\vec{E} = \left( k \frac{|Q_1|}{r_1^2} + k \frac{|Q_2|}{r_2^2} \right) \hat{i} \quad (2)
\]

\[
\vec{E} = \left( 9 \times 10^9 \left[ \frac{4 \times 10^{-6}}{6^2} + \frac{2 \times 10^{-6}}{1^2} \right] \right) \hat{i} \quad (3)
\]

\[
\vec{E} = \left( 9 \times 10^9 \left[ 111 \times 10^{-9} + 2 \times 10^{-6} \right] \right) \hat{i} \quad (4)
\]

\[
\vec{E} = \left( 9 \times 10^9 \left[ 2 \times 10^{-6} \right] \right) \hat{i} \quad (5)
\]

\[
\vec{E} = \left( 19 \times 10^3 \right) \frac{V}{m} \hat{i} \quad (6)
\]

Please note that since we took care of the directions of the electric field “by hand”, that we used the absolute value of the charges so that we would not accidentally account for the signs of the charges twice.

(b) Find the magnitude of the potential at the origin. This is easy since potential is a scalar (no vectors!)

\[
V = k \frac{Q_1}{r_1} + k \frac{Q_2}{r_2} \quad (7)
\]

\[
V = 9 \times 10^9 \left[ \frac{4 \times 10^{-6}}{6} + \frac{-2 \times 10^{-6}}{1} \right] \quad (8)
\]
\[ V = 9 \times 10^9 \left[-1.33 \times 10^{-6}\right] \] (9)
\[ V = 11970V \] (10)

(c) Find the place on the x axis at which the field is zero.

OK, this is a tricky one. First we look between the two charges. Well, in between they both point their field to the right. This means that these two fields will never cancel. So we know that the zero point is to the left of the left charge or to the right of the right charge since only at these places are the directions of the fields due to the two charges opposite. Now, we know that to be equal in strength, we need to be close to the weak charge and far from the strong charge. This only happens when we are to the right of the right charge. OK, in this region, we can calculate the results:

\[ |\vec{E}| = 0 = k \frac{4 \times 10^{-6}}{(x + 6)^2} - \frac{2 \times 10^{-6}}{(x - 1)^2} \] (11)
\[ 0 = k \times 10^{-6} \left[ \frac{4}{(x + 6)^2} - \frac{2}{(x - 1)^2} \right] \] (12)
\[ 0 = \left[ \frac{4}{(x + 6)^2} - \frac{2}{(x - 1)^2} \right] \] (13)
\[ \frac{4}{(x + 6)^2} = \frac{2}{(x - 1)^2} \] (14)
\[ 4 \times (x - 1)^2 = 2 \times (x + 6)^2 \] (15)
\[ 4 \times (x^2 - 2x + 1) = 2 \times (x^2 + 12x + 36) \] (16)
\[ 4x^2 - 8x + 4 = 2x^2 + 24x + 72 \] (17)
\[ 2x^2 - 32x - 68 = 0 \] (18)
\[ x^2 - 16x - 34 = 0 \] (19)
\[ x = 17.9m \] (20)

(d) Find the places on the x axis at which the potential is zero.

Now, with potential, the quantity is a scalar. This means that the positive charge always contributes a positive amount, and the negative charge always contributes a negative amount. All you need to satisfy is that you get the same size contribution from each and the potential will be zero. This occurs when you are closer to the smaller charge and happens both in between the charges and to the right of the small charge.

In Between:

\[ 0 = k \frac{Q_1}{(x + 6)} - k \frac{Q_2}{(1 - x)} \] (21)
\[ k \frac{Q_1}{(x + 6)} = k \frac{Q_2}{(1 - x)} \] (22)
\[
\frac{4}{x + 6} = \frac{2}{1 - x} \quad (23)
\]

\[
4 \times (1 - x) = 2 \times (x + 6) \quad (24)
\]

\[
4 - 4x = 2x + 12 \quad (25)
\]

\[
0 = 6x + 8 \quad (26)
\]

\[
x = -1.333m \quad (27)
\]

To the Right:

\[
0 = k \frac{Q_1}{x + 6} - k \frac{Q_2}{x - 1} \quad (28)
\]

\[
k \frac{Q_1}{x + 6} = k \frac{Q_2}{x - 1} \quad (29)
\]

\[
4 \frac{x + 6}{x - 1} = \frac{2}{x - 1} \quad (30)
\]

\[
4 \times (x - 1) = 2 \times (x + 6) \quad (31)
\]

\[
4x - 4 = 2x + 12 \quad (32)
\]

\[
2x - 16 = 0 \quad (33)
\]

\[
x = 8.0m \quad (34)
\]
2. Shown in the figure below are two point charges located along the x axis. Determine all the following at the point $x = 0$ and $y = 8$ m:

![Diagram of two point charges along the x axis with y axis labeled and distances marked 6m.]

(a) The electric field.
Again, this is the hard(er) one since the electric field is a vector. However, there is some simplification to the problem since we understand that the answer will lie along the y axis by symmetry. So, all we have to calculate is the $y$ component of each field. First, the magnitude of the field:

$$|\vec{E}| = k \frac{Q}{r^2}$$

$$r = \sqrt{6^2 + 8^2} = 10$$

$$|\vec{E}| = 9 \times 10^9 \frac{4 \times 10^{-6}}{10^2}$$

$$|\vec{E}| = 360 \text{ V/m}$$

To get the $y$ component, we need to multiply this by the $\cos \theta$. Well, since we have all sides of the triangle we can get this easily:

$$\cos \theta = \frac{adj}{hyp} = \frac{8}{10} = 0.8$$

$$E_y(\text{one charge}) = |\vec{E}| \cos \theta$$

$$E_y(\text{one charge}) = 360 \times 0.8 = 288 \text{ V/m}$$

$$E_y(\text{two charges}) = 2 \times E_y(\text{one charge}) = 576 \text{ V/m}$$

(b) The electric potential.
As always, no vectors, no problem. We just add up the scalar potentials:

$$V = k \frac{Q}{r} + k \frac{Q}{r}$$

$$V = 2k \frac{Q}{r}$$
\[ V = 2 \times 9 \times 10^9 \frac{4 \times 10^{-6}}{10} \]
\[ V = 7200V \]
3. Point Charges

(a) Find the magnitude and direction of the electric field, \( \mathbf{E} \) at the point \( \mathbf{P} \).

To solve this problem we will need to add the electric fields from each of the two point charges, \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \). We will need to write these in component format so that they can be added to produce the final result. First, however, let us find the magnitude of each field:

\[
|\mathbf{E}_1| = k\frac{Q_1}{r_{1P}^2},
\]

(47)

\[
|\mathbf{E}_1| = 9 \times 10^6 \frac{4\mu C}{3^2 + 4^2}.
\]

(48)

\[
|\mathbf{E}_1| = 9 \times 10^6 \frac{4 \times 10^{-6}}{41}\]

(49)

\[
|\mathbf{E}_1| = 878 \frac{V}{m}
\]

(50)

\[
|\mathbf{E}_2| = k\frac{Q_2}{r_{2P}^2},
\]

(51)

\[
|\mathbf{E}_2| = 9 \times 10^6 \frac{2\mu C}{2^2 + 5^2}.
\]

(52)

\[
|\mathbf{E}_2| = 9 \times 10^6 \frac{2 \times 10^{-6}}{29}\]

(53)

\[
|\mathbf{E}_2| = 621 \frac{V}{m}
\]

(54)

OK, now we have the magnitude of each of these two fields, but these are vectors and not scalars, so we need to add them as vectors. Using the angles labeled in the figures we can find the following:

\[
\mathbf{E}_1 = |\mathbf{E}_1| \cos \theta_1 \hat{i} + |\mathbf{E}_1| \sin \theta_1 \hat{j}
\]

(55)

\[
\cos \theta_1 = \frac{5}{\sqrt{3^2 + 4^2}} = 0.78087
\]

(56)

\[
\sin \theta_1 = \frac{4}{\sqrt{3^2 + 4^2}} = 0.62470
\]

(57)

\[
\mathbf{E}_1 = 878 \times 0.78087 \hat{i} + 878 \times 0.62470 \hat{j}
\]

(58)

\[
\mathbf{E}_1 = 686 \frac{V}{m} \hat{i} + 548 \frac{V}{m} \hat{j}
\]

(59)

\[
\mathbf{E}_2 = -|\mathbf{E}_2| \cos \theta_2 \hat{i} - |\mathbf{E}_2| \sin \theta_2 \hat{j}
\]

(60)
\[
\begin{align*}
\cos \theta_2 &= \frac{2}{\sqrt{2^2 + 5^2}} = 0.37139 \\
\sin \theta_2 &= \frac{5}{\sqrt{2^2 + 5^2}} = 0.92848 \\
\vec{E}_2 &= -621 \times 0.37139 \hat{i} - 621 \times 0.92848 \hat{j} \\
\vec{E}_2 &= -231 \frac{V}{m} \hat{i} - 576 \frac{V}{m} \hat{j}
\end{align*}
\]

Now that each of these is in component format, they are very easy to add:

\[
\begin{align*}
\vec{E}_{TOT} &= \vec{E}_1 + \vec{E}_2 \\
\vec{E}_{TOT} &= (686 - 231) \hat{i} + (548 - 576) \hat{j} \\
\vec{E}_{TOT} &= 455 \frac{V}{m} \hat{i} - 28 \frac{V}{m} \hat{j}
\end{align*}
\]

Then, we simply get the magnitude and direction by:

\[
\begin{align*}
|\vec{E}_{TOT}| &= \sqrt{455^2 + 28^2} \\
\vec{E}_{TOT} &= 456 \frac{V}{m} \\
\tan \theta_{TOT} &= -\frac{28}{455} \\
\theta_{TOT} &= -3.5^\circ
\end{align*}
\]

(b) What would be the magnitude of the force on a particle of charge \( q = 4 \mu C \) placed at point P:

This one is REALLY easy since \( \vec{F} = q \vec{E} \). So, using the total field from the previous step, we can easily get the total force:

\[
\begin{align*}
|\vec{F}| &= 4 \times 10^{-6} \times 456 \\
|\vec{F}| &= 0.1824 \text{N}
\end{align*}
\]

(c) What is the potential, \( V \), at the point P?

This one is much easier to calculate than the field simply because the potential is a scalar and the field is a vector. We calculate it in two parts:

\[
\begin{align*}
V_1 &= k \frac{Q_1}{r_{1P}} \\
V_1 &= 9 \times 10^9 \frac{4 \times 10^{-6}}{\sqrt{3^2 + 4^2}} \\
V_1 &= 5622 \text{V o lts}
\end{align*}
\]
\[ V_2 = k \frac{Q_2}{r_{2P}} \]  
(77)

\[ V_2 = 9 \times 10^9 \frac{-2 \times 10^{-6}}{\sqrt{2^2 + 5^2}} \]  
(78)

\[ V_2 = -3343 \text{ Volts} \]  
(79)

\[ V_{TOT} = V_1 + V_2 = 2279 \text{V} \]  
(80)

(d) How much work is required to bring the \( q = 4 \mu C \) charge from infinity to the point \( P \).

Ah ha! Again an easy one. The particle has a total energy of \( U = qV(P) \) when placed at the point \( P \). We solved for the potential at point \( P \) \( (V(P) = 2279 \text{Volts}) \) in the previous step. So, all we do is multiply:

\[ U = qV \]  
(81)

\[ U = 4 \times 10^{-6} \times 2279 \]  
(82)

\[ U = W = 9.12 \text{mJ} \]  
(83)

(e) Assume that the \( q = 4 \mu C \) charge has a mass of \( m = 12 \mu g \) and is released from rest starting at the point \( P \). What will be the velocity of this mass when it reaches infinity?

Again, very easy. All of the 9.12 milli-Joules will turn into kinetic energy when the particle is released and moves to infinity. So we find:

\[ \frac{1}{2}mv^2 = U \]  
(84)

\[ v = \sqrt{\frac{2U}{m}} \]  
(85)

\[ v = \sqrt{\frac{2 + 9 \times 10^{-3}}{12 \times 10^{-9}}} \]  
(86)

\[ v = 1.23 \times 10^3 \frac{m}{s} \]  
(87)
4. Shown in the figure below is a sphere of charge \( +Q \) and radius \( a \). Use Gauss Law to find the electric field in each of these regions:

(a) Region I \( (r > a) \).

OK, the Gauss surface for a spherical charge will be a sphere!!!

\[
EA = \frac{Q_{\text{in}}}{\varepsilon_0} \quad (88)
\]
\[
A = 4\pi r^2 \quad (89)
\]
\[
Q_{\text{in}} = Q \quad (90)
\]
\[
E4\pi r^2 = \frac{Q}{\varepsilon_0} \quad (91)
\]
\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \quad (92)
\]

(b) Region II \( (r < a) \).

This one is a little harder since we only contain a fraction of the total charge inside our sphere:

\[
EA = \frac{Q_{\text{in}}}{\varepsilon_0} \quad (93)
\]
\[
A = 4\pi r^2 \quad (94)
\]
\[
Q_{\text{in}} = Q \frac{V_{\text{taken}}}{V_{\text{total}}} \quad (95)
\]
\[
Q_{\text{in}} = Q \frac{4}{3}\pi a^3 \quad (96)
\]
\[
Q_{\text{in}} = Q \frac{r^3}{a^3} \quad (97)
\]
\[
E4\pi r^2 = \frac{Q_{\text{in}}}{\varepsilon_0} \quad (98)
\]
\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{a^3} \quad (99)
\]
5. Shown in the figure below is an infinitely long cylinder of radius \( a \) filled with a uniform change density \( +\rho \text{Coulombs/m}^2 \). Use Gauss Law to find the electric field in each of these regions:

(a) Region I \((r > a)\).

In this problem we have an infinite object. To have a non-zero field, an infinite object must have an infinite total charge. This is why we DO NOT deal with the total charge of the object, we only deal with charge density!! Also, since we will have to make a closed Gaussian surface, we will need to make this finite. Doing so will introduce a non-physical parameter into the calculation. This non-physical parameter will cancel in the answer (unless we make a mistake).

We choose as out Gaussian surface, a cylinder with radius \( r \) and length \( L \). Note that only the barrel of the cylinder counts for Gauss Law since the electric field is along the surface at the end caps.

\[
E A = \frac{Q_{\text{in}}}{\varepsilon_0} \quad (100)
\]

\[
A = 2\pi r L \quad (101)
\]

\[
Q_{\text{in}} = \rho \times V_{\text{taken}} = \rho \times \pi a^2 L \quad (102)
\]

\[
E 2\pi r L = \frac{\rho \times \pi a^2 L}{\varepsilon_0} \quad (103)
\]

\[
E 2r = \frac{\rho \times a^2}{\varepsilon_0} \quad (104)
\]

\[
E = \frac{\rho \times a^2}{2\varepsilon_0} \frac{1}{r} \quad (105)
\]

(b) Region II \((r < a)\). This time the derivation is very similar. The only difference is the volume taken:

\[
E A = \frac{Q_{\text{in}}}{\varepsilon_0} \quad (106)
\]

\[
A = 2\pi r L \quad (107)
\]

\[
Q_{\text{in}} = \rho \times V_{\text{taken}} = \rho \times \pi r^2 L \quad (108)
\]

\[
E 2\pi r L = \frac{\rho \times \pi r^2 L}{\varepsilon_0} \quad (109)
\]
\[ E_{2r} = \frac{\rho \times r^2}{\varepsilon_0} \quad \text{(110)} \]
\[ E = \frac{\rho}{2\varepsilon_0} r \quad \text{(111)} \]
6. Shown in the figure below are two concentric spheres of charge. A charge +Q is uniformly distributed over the outer sphere between r=b and r=c. A charge -Q is uniformly distributed over the inner sphere r < a. The space between the spheres, a < r < b, is empty. Use Gauss Law to find the electric field in each of these regions:

(a) Region I (r > c).

SO EASY! $Q_{in} = 0$, thus $E=0$.

(b) Region II (b < r < c).

OK, now we have a challenge (the biggest one of this problem). In this case, the $Q_{in}$ will include all of the inner charge and only a fraction of the outer charge. YUCH!!!

\[
EA = \frac{Q_{in}}{\epsilon_0} \quad (112)
\]

\[
A = 4\pi r^2 \quad (113)
\]

\[
Q_{in} = Q \times \frac{V_{taken}}{V_{total}} - Q \quad (114)
\]

\[
V_{total} = 4\pi c^3 - \frac{4}{3}\pi b^3 \quad (115)
\]

\[
V_{total} = 4\pi r^3 - \frac{4}{3}\pi b^3 \quad (116)
\]

\[
\frac{V_{taken}}{V_{total}} = \frac{\frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3}{\frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3} \quad (117)
\]

\[
\frac{V_{taken}}{V_{total}} = \frac{r^3 - b^3}{c^3 - b^3} \quad (118)
\]

\[
Q_{in} = Q_c^3 - b^3 - Q = Q_c^3 - c^3 \quad (119)
\]
\[
E 4\pi r^2 = \frac{Q}{\epsilon_0} \frac{r^3 - b^3}{c^3 - b^3} \quad (120)
\]
\[
E = \frac{Q}{4\pi \epsilon_0} \frac{r^3 - b^3}{c^3 - b^3} \frac{1}{r^2} \quad (121)
\]

(c) Region III \((a < r < b)\).

Oh, sweet simplicity!!! We take all the inner charge and none of the outer charge. So, \(Q_{\text{in}} = -Q\).

\[
EA = \frac{Q_{\text{in}}}{\epsilon_0} \quad (122)
\]
\[
A = 4\pi r^2 \quad (123)
\]
\[
Q_{\text{in}} = -Q \quad (124)
\]
\[
E 4\pi r^2 = \frac{-Q}{\epsilon_0} \quad (125)
\]
\[
E = - \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} \quad (126)
\]

Hmm...seem familiar...YES, this is the same as two problems ago.

(d) Region IV \((r < a)\).

Deja vu...This is the same as two problems ago....

\[
EA = \frac{Q_{\text{in}}}{\epsilon_0} \quad (127)
\]
\[
A = 4\pi r^2 \quad (128)
\]
\[
Q_{\text{in}} = -Q \frac{V_{\text{taken}}}{V_{\text{total}}} \quad (129)
\]
\[
Q_{\text{in}} = -Q \frac{4\pi r^3}{3\pi a^3} \quad (130)
\]
\[
Q_{\text{in}} = -Q \frac{r^3}{a^3} \quad (131)
\]
\[
E 4\pi r^2 = \frac{-Q}{\epsilon_0} \frac{r^2}{a^3} \quad (132)
\]
\[
E = - \frac{1}{4\pi \epsilon_0} \frac{Q}{a^3} r \quad (133)
\]

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7. Shown in the figure below are two concentric cylinders of charge. The cylinders are uniformly filled with charge densities \( \pm \rho_{\text{Cylindrical}} \) as indicated in the figure. The space between the cylinders, \( a < r < b \), is empty. Use Gauss Law to find the electric field in each of these regions:

(a) Region I (\( r > c \)).

OK, you know the drill...cylindrical charge cylindrical surface. The pain here is the volume of the outer cylinder.

\[
EA = \frac{Q_{\text{in}}}{\epsilon_0} \tag{134}
\]

\[
A = 2\pi r L \tag{135}
\]

\[
Q_{\text{in}} = Q_{\text{outer}} + Q_{\text{inner}} \tag{136}
\]

\[
Q_{\text{inner}} = -\rho \times V_{\text{inner}} \tag{137}
\]

\[
Q_{\text{inner}} = -\rho \times \pi a^2 L \tag{138}
\]

\[
Q_{\text{outer}} = \rho \times V_{\text{outer}} \tag{139}
\]

\[
Q_{\text{outer}} = \rho \times [\pi c^2 L - \pi b^2 L] \tag{140}
\]

\[
Q_{\text{outer}} = \rho \pi L [c^2 - b^2] \tag{141}
\]

\[
Q_{\text{in}} = \rho \pi L [c^2 - b^2] - \rho \times \pi a^2 L \tag{142}
\]

\[
Q_{\text{in}} = \rho \pi L [c^2 - b^2 - a^2] \tag{143}
\]

\[
E 2\pi r L = \frac{\rho \pi L [c^2 - b^2 - a^2]}{\epsilon_0} \tag{144}
\]

\[
E 2r = \rho \frac{[c^2 - b^2 - a^2]}{\epsilon_0} \tag{145}
\]
\[ E2r = \rho \frac{[c^2 - b^2 - a^2]}{2\varepsilon_0} \frac{1}{r} \]  

(b) Region II \((b < r < c)\).

It is always darkest before the dawn. In this part of the problem, we take all of the inner charge, but only a fraction of the outer charge.

\[ EA = \frac{Q_{in}}{\varepsilon_0} \]  
\[ A = 2\pi rL \]  
\[ Q_{in} = Q_{outer} + Q_{inner} \]  
\[ Q_{inner} = -\rho \times V_{inner} \]  
\[ Q_{inner} = -\rho \times \pi a^2 L \]  
\[ Q_{outer} = \rho \times V_{taken} \]  
\[ Q_{outer} = \rho \times [\pi r^2 L - \pi b^2 L] \]  
\[ Q_{in} = \rho \pi L [r^2 - b^2] - \rho \times \pi a^2 L \]  
\[ Q_{in} = \rho \pi L [r^2 - b^2 - a^2] \]  
\[ E2\pi rL = \frac{\rho \pi L [r^2 - b^2 - a^2]}{\varepsilon_0} \]  
\[ E2r = \rho \frac{[r^2 - b^2 - a^2]}{\varepsilon_0} \frac{1}{r} \]  
\[ E2r = \rho \frac{[r^2 - b^2 - a^2]}{2\varepsilon_0} \frac{1}{r} \]  

(c) Region III \((a < r < b)\).

Dawn. This is just like part a of a previous problem.

\[ EA = \frac{Q_{in}}{\varepsilon_0} \]  
\[ A = 2\pi rL \]  
\[ Q_{in} = -\rho \times V_{taken} = \rho \times \pi a^2 L \]  
\[ E2\pi rL = -\frac{\rho \times \pi a^2 L}{\varepsilon_0} \]  
\[ E2r = -\frac{\rho \times a^2}{\varepsilon_0} \]  
\[ E = -\frac{\rho \times a^2}{2\varepsilon_0} \frac{1}{r} \]  

(d) Region IV \((r < a)\).

Dawn. This is just like part b of a previous problem.

\[ EA = \frac{Q_{in}}{\varepsilon_0} \]  

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\[ A = 2\pi r L \] (167)

\[ Q_{\text{in}} = -\rho \times V_{\text{taken}} = \rho \times \pi r^2 L \] (168)

\[ E \frac{2\pi r L}{\varepsilon_0} = -\rho \times \pi r^2 L \] (169)

\[ E \frac{2r}{\varepsilon_0} = -\rho \times r^2 \] (170)

\[ E = -\frac{\rho}{2\varepsilon_0} r \] (171)
8. Shown in the figure below are two concentric *CONDUCTING* spheres of charge. Because the spheres are conducting, the charge will not be uniformly distributed over their volumes, but will instead accumulate on the surfaces. The inner sphere has a total charge -Q and the outer sphere has a total charge +2Q. Use Gauss Law to determine all the following:

![Diagram of concentric spheres](image)

(a) The electric field in Region I (r > c).

Dude, conducting spheres are way easy. The charge is like all like on the surfaces. So like your Gauss surface always either contains some charge of contains none (no bogus fractions). Also, E is zero inside conductors, sweet. So check it, determine the charges first, then the fields. So, all your answers are point charge answer with like the right charge.

Total charge = +1Q,

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \tag{172} \]

(b) The electric field in Region II (b < r < c).

IN THE CONDUCTOR, E=0.

(c) The electric field in Region III (a < r < b).

Total charge = -1Q,

\[ E = -\frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \tag{173} \]

(d) The electric field in Region IV (r < a).

IN THE CONDUCTOR, E=0.

(e) The charge on the surface r=c.

+Q (since you needed to send +1Q for shielding, that leaves +1Q here).
(f) The charge on the surface r=b.
   +Q (since you like gotta shield out the inner guys charge to have zero field in the conductor).

(g) The charge on the surface r=a.
   -Q (all the inner charge).
9. The figure below shows a set of equipotential lines that were created by some set of charges. The lines represent potentials ranging from -300 V to +300 V in steps of 100 V. Also labelled in the plot are the points a = (0,0), b = (1.5 m, 3 m), and c = (3 m, 2.5 m).

(a) Which point (a, b, or c) has the highest \( \vec{E} \)?
   a

(b) Which point (a, b, or c) has the lowest \( \vec{E} \)?
   c

(c) Estimate the strength of the electric field at a point 1/2 way between b and c.
   These points are separated by 100 V. They are also separated by some distance. Field is volts per meter...

\[
\Delta V = 100 \\
d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \\
d = \sqrt{(1.5)^2 + (0.5)^2} \\
d = 1.58 \\
E = \frac{V}{d} = 63.24 \text{V/m} 
\]

(d) Draw several (roughly 6) electric field lines on the plot.

(e) If these lines were generated by two point charges (one to the left and one to the right), determine which side of the graph contains which charge (i.e., either +left/-right or -left/+right).
   -left/+right
10. Shown in the figure below is a parallel plate capacitor. The area of each plate is 1 $m^2$ and the separation between the plates is 1 mm (0.001 m). The region between the plates is filled with a dielectric material of dielectric constant $K=2$.

(a) Determine the capacitance of this device.

OH NO!!! A dielectric. Don't panic. All we do is use $\epsilon = K\epsilon_0$ in every place we would have used $\epsilon_0$.

$$C = \epsilon \frac{A}{d} \quad (180)$$

$$C = 2\epsilon_0 \frac{1}{0.001} \quad (181)$$

$$C = 2 \times 8.85 \times 10^{-12} \frac{1}{0.001} \quad (182)$$

$$C = 17.7 \times 10^{-9} F \quad (183)$$
(b) If the capacitor is charged to \( Q = 6\mu C \), when is the voltage difference between the plates?

\[
V = \frac{Q}{C} \tag{184}
\]

\[
V = \frac{6 \times 10^{-6}}{17.7 \times 10^{-9}} \tag{185}
\]

\[
V = 339 \text{Volts} \tag{186}
\]

(c) What is the electric field between the plates?

\[
E = \frac{\Delta V}{\Delta x} = \frac{339}{0.001} = 33900 \text{V/m} \tag{187}
\]

(d) What is the stored energy in the system?

\[
U = \frac{1}{2} QV \tag{189}
\]

\[
U = \frac{1}{2} (6 \times 10^{-6}) 339 = 0.001 \text{J} \tag{190}
\]

(e) In what form is the energy stored? Electric Field
11. Shown in the figure below is a mass analyzer system. This system operates by accelerating a charged particle across the gap between two capacitor plates, giving it energy. After it has been accelerated, it enters a magnetic field and is bent in a circular trajectory until it reaches the exit.

You are to design the mass analyzer system so that $^{12}$C ions with charge, $q = +1.6 \times 10^{-19}$, and mass $m = 12 \times 1.67 \times 10^{-27} \text{kg} = 2.00 \times 10^{-26} \text{kg}$ are accelerated to an energy of 25 MeV. Answer all the following:

(a) What should be the voltage difference across the capacitor?

This one is very easy if you are good in the use of eV and MeV units. The charge of the carbon ion in this problem is 1 e (or $1.602 \times 10^{-19} \text{C}$). First, let's solve this problem using e's as the charges:

\[
U = qV \quad (192)
\]
\[
25 \text{MeV} = (1e)V \quad (193)
\]
\[
25MV = V \quad (194)
\]
\[
V = 25,000,000 \text{Volts}. \quad (195)
\]

OK, if you really like Joules, we can do it that way also:

\[
U = 25 \text{MeV} \times \frac{1.602 \times 10^{-19} \text{C}}{1e} = 4.005 \times 10^{-12} \text{J} \quad (196)
\]
\[
4.005 \times 10^{-12} \text{J} = qV \quad (197)
\]
\[
4.005 \times 10^{-12} \text{J} = 1.602 \times 10^{-19} \text{CV} \quad (198)
\]
\[
V = 25,000,000 \text{Volts} \quad (199)
\]

(b) If the separation between the plates is 1 meter, what would be the electric field between the plates?

Here the simplest way to remember the solution is to realize that the electric field has units of $\frac{\text{V}}{\text{m}}$. The full formula uses a derivative of teh potential to make each of the vector components of E, $(E_x = -\frac{\Delta V}{\Delta x})$. However, for a constant electric field (like in the capacitor plates, the formula is simpler $(E_x = -\frac{\Delta V}{\Delta x})$). Thus,

\[
E_x = -\frac{\Delta V}{\Delta x} \quad (200)
\]
\[
E_x = \frac{\Delta V}{\Delta x} = \frac{\Delta V}{1\text{meter}} \quad (201)
\]
\[
E_x = 25,000,000 \frac{\text{V}}{m} \quad E_y = E_z = 0 \quad (202)
\]
(c) If the area of each plate is $50m^2$, what is the capacitance and charge of this device?

For a simple parallel plate capacitor, the capacitance is given by:

$$C = \frac{\varepsilon_0 A}{d} = \varepsilon_0 \frac{50m^2}{1\text{meter}}$$  \hspace{1cm} (203)

$$C = 8.854 \times 10^{-12}\frac{F}{m} \frac{50m^2}{1\text{meter}}$$  \hspace{1cm} (204)

$$C = 4.427 \times 10^{-10}F = 443pF$$  \hspace{1cm} (205)

OK, for any capacitor, the voltage across the capacitor is proportional to the charge it holds as given by:

$$V = \frac{Q}{C}$$  \hspace{1cm} (206)

$$25,000,000 = \frac{Q}{443 \times 10^{-12}}$$  \hspace{1cm} (207)

$$Q = 0.01107\text{Coulombs} = 11.07\text{mC}$$  \hspace{1cm} (208)

(d) What will be the velocity of the ions after they are accelerated by the capacitor?

Well, we are told the energy of the ions when they leave the capacitor (25 MeV). All this energy is in the form of Kinetic Energy. Since this problem is non-relativistic:

$$\frac{1}{2}mv^2 = 25\text{MeV} = 25 \times 10^6\text{eV} \frac{1.602 \times 10^{-19}C}{1e}$$  \hspace{1cm} (209)

$$\frac{1}{2}mv^2 = 4.005 \times 10^{-12}J$$  \hspace{1cm} (210)

$$v = \sqrt{\frac{2 \times 4.005 \times 10^{-12}J}{m}}$$  \hspace{1cm} (211)

$$v = \sqrt{\frac{2 \times 4.005 \times 10^{-12}J}{2.00 \times 10^{-20}\text{kg}}}$$  \hspace{1cm} (212)

$$v = 2 \times 10^7\text{m/s}\text{.}$$  \hspace{1cm} (213)

(e) What magnetic field should be used so that the ions travel in a circular path with radius, $R = 1.2\text{meters}$.

The force on the ion on the magnetic field is given by $F = qvB$. Thus:

$$F = qvB = ma$$  \hspace{1cm} (214)
\[ a_c = \frac{v^2}{r} \]  
(215)

\[ qvB = m \frac{v^2}{r} \]  
(216)

\[ B = \frac{mv}{qr} \]  
(217)

\[ B = \frac{2.00 \times 10^{-26} \text{kg} \times 2 \times 10^7}{1.602 \times 10^{-19} \text{C}} \]  
(218)

\[ B = 2.08 \text{ Tesla} \]  
(219)

(f) How would this differ if the ions were instead $^{14}_6$C ions? Answer in a few sentences, not a calculation.

If the ions were $^{14}_6$C, the energy would not change (since mass did not enter those calculations). However, the ions would be harder to bend and move with a larger radius in the magnetic field.

(g) What would be the usefulness of this device?

You could use it to measure both the $^{14}_6$C and $^{12}_6$C content of some sample of once-living material and thereby date the material.

(h) What electric field would you put inside the magnet so as to make the ions travel in a straight line instead of a curved path?

In this case, we want an electric field strong enough that it’s force cancels out the magnetic force:

\[ qE = qvB \]  
(220)

\[ E = vB \]  
(221)

\[ E = 2 \times 10^7 \times 2.08 = 4.16 \times 10^7 \frac{V}{m} \]  
(222)
12. Shown below is a circuit containing a resistor network. Let the values of
the labelled components be as follows: \( R_1 = 2k\Omega, R_2 = 3k\Omega, R_3 = 4k\Omega, R_4 = 6k\Omega, V = 10V \)

(a) Find the equivalent resistance of the network.

To solve this problem we must reduce the resistor network step by step. Three figures were generated to show this reduction. In the first figure, we combine resistors \( R_2 \) and \( R_3 \) to form an equivalent resistor \( R_A \).

The calculation of \( R_A \) is as follows:

\[
\frac{1}{R_A} = \frac{1}{R_2} + \frac{1}{R_3} \quad (223)
\]

\[
\frac{1}{R_A} = \frac{1}{3k\Omega} + \frac{1}{4k\Omega} \quad (224)
\]

\[
\frac{1}{R_A} = 0.5\overline{833333} \frac{1}{k\Omega} \quad (225)
\]

\[
R_A = 1.714k\Omega \quad (226)
\]

Next we reduce the circuit further by combining resistor \( R_1 \) with resistor \( R_A \) and thereby form a resistor that we will call \( R_B \). This one is a simpler calculation:

\[
R_B = R_1 + R_A \quad (227)
\]

\[
R_B = 2k\Omega + 1.714k\Omega \quad (228)
\]

\[
R_B = 3.714k\Omega \quad (229)
\]
Finally, we need to combine resistors $R_B$ and $R_4$ so as to make an equivalent resistor $R_C$. This will end the reduction and answer this part of the problem:

\[
\frac{1}{R_C} = \frac{1}{R_B} + \frac{1}{R_4} \quad (230)
\]
\[
\frac{1}{R_C} = \frac{1}{3.714k\Omega} + \frac{1}{6k\Omega} \quad (231)
\]
\[
\frac{1}{R_C} = 0.436 \frac{1}{k\Omega} \quad (232)
\]
Find the current through each resistor.

OK, to solve all of the remaining steps of this problem, we will use a technique that is the equivalent of filling a table. The table will have 7 rows labelled as \((R_1, R_2, R_3, R_4, R_A, R_B, R_C)\). There will be four columns labelled as \((R, V, I, P)\). At this point, the table can be filled in as follows:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (Å)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_2)</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_3)</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_4)</td>
<td>6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_A)</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_B)</td>
<td>3714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_C)</td>
<td>2294</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OK, now here is the secret to solving the table. Since \(V = IR\), whenever, we get any two of the three values \((V, I, R)\) we can get the third. In the bottom row, we can find \(I\) of resistor \(C\) since we have \(R\) and \(V\). After that \(P = VI\), and we have finished filling in the bottom row:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (Å)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_2)</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_3)</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_4)</td>
<td>6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_A)</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_B)</td>
<td>3714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_C)</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

OK, now for the cool part. The resistor \(R_C\) is made from the parallel combination of \(R_4\) and \(R_B\). Since this is a parallel combination, the voltages in \(R_C\), \(R_4\), and \(R_B\), are ALL THE SAME! So we simply copy the voltage on \(R_C\) to \(R_4\) and \(R_B\) giving us:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (Å)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_2)</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_3)</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_4)</td>
<td>6000</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_A)</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_B)</td>
<td>3714</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
<tr>
<td>(R_C)</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

OK, now we see that we have 2 out of 3 for \(R_4\), and \(R_B\), and so we can fill in the full lines giving us:
<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₄</td>
<td>6000</td>
<td>10</td>
<td>0.0016666</td>
<td>0.016666</td>
</tr>
<tr>
<td>R₅</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₆</td>
<td>3714</td>
<td>10</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>R₇</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

Fine, now we know that $R_B$ was made from the series combination of $R_1$ and $R_A$. In that case, $R_1$, $R_A$, and $R_B$ all have identically the same current, so we copy that value to multiple places:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₄</td>
<td>6000</td>
<td>10</td>
<td>0.0016666</td>
<td>0.016666</td>
</tr>
<tr>
<td>R₅</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₆</td>
<td>3714</td>
<td>10</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>R₇</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

Cool, we have 2 out of 3 in the rows for resistors $R_1$ and $R_A$ and so we fill out these lines to have:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₄</td>
<td>6000</td>
<td>10</td>
<td>0.0016666</td>
<td>0.016666</td>
</tr>
<tr>
<td>R₅</td>
<td>1714</td>
<td>4.61</td>
<td>0.00269</td>
<td>0.0124</td>
</tr>
<tr>
<td>R₆</td>
<td>3714</td>
<td>10</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>R₇</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

OK gang, last step! $R_A$ is made from the parallel combination of $R_2$ and $R_3$. So, these all have the same voltage that we copy to other parts of the table:

<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>2000</td>
<td>5.38</td>
<td>0.00269</td>
<td>0.0145</td>
</tr>
<tr>
<td>R₂</td>
<td>3000</td>
<td>4.61</td>
<td>0.00269</td>
<td>0.0124</td>
</tr>
<tr>
<td>R₃</td>
<td>4000</td>
<td>4.61</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>R₄</td>
<td>6000</td>
<td>10</td>
<td>0.0016666</td>
<td>0.016666</td>
</tr>
<tr>
<td>R₅</td>
<td>1714</td>
<td>4.61</td>
<td>0.00269</td>
<td>0.0124</td>
</tr>
<tr>
<td>R₆</td>
<td>3714</td>
<td>10</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>R₇</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

Finally 2 out of 3 in the last two rows and we simply finish out the table:
<table>
<thead>
<tr>
<th></th>
<th>R (Ω)</th>
<th>V (Volts)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>2000</td>
<td>5.38</td>
<td>0.00269</td>
<td>0.0145</td>
</tr>
<tr>
<td>$R_2$</td>
<td>3000</td>
<td>4.61</td>
<td>0.0015366</td>
<td>0.00708</td>
</tr>
<tr>
<td>$R_3$</td>
<td>4000</td>
<td>4.61</td>
<td>0.0011525</td>
<td>0.00531</td>
</tr>
<tr>
<td>$R_4$</td>
<td>6000</td>
<td>10</td>
<td>0.0016666</td>
<td>0.016666</td>
</tr>
<tr>
<td>$R_A$</td>
<td>1714</td>
<td>4.61</td>
<td>0.00269</td>
<td>0.0124</td>
</tr>
<tr>
<td>$R_B$</td>
<td>3714</td>
<td>10</td>
<td>0.00269</td>
<td>0.0269</td>
</tr>
<tr>
<td>$R_C$</td>
<td>2294</td>
<td>10</td>
<td>0.00436</td>
<td>0.0436</td>
</tr>
</tbody>
</table>

(c) Find the voltage across each resistor.
   in table

(d) Find the power dissipated by each resistor.
   in table
   Of course, you only need to write the table once.
13. Shown below is a circuit containing a capacitor network. Let the values of the labelled components be as follows: \( C_1 = 2\mu F, C_2 = 3\mu F, C_3 = 4\mu F, C_4 = 6\mu F, V = 10V \)

(a) Find the equivalent capacitance of the network.

OK gang. This one looks pretty similar to the previous. The major difference is that time we are dealing with capacitors instead of resistors and so the methods of adding them in parallel and series are different.

The first figure shows how we make the equivalent capacitor \( C_A \) from \( C_2 \) and \( C_3 \) as follows:

\[
C_A = C_2 + C_3 \quad \text{(234)}
\]

\[
C_A = 3\mu F + 4\mu F = 7\mu F \quad \text{(235)}
\]

Now we must combine \( C_A \) and \( C_1 \) in order to form \( C_B \):

\[
\frac{1}{C_B} = \frac{1}{C_1} + \frac{1}{C_A} \quad \text{(236)}
\]

\[
\frac{1}{C_B} = \frac{1}{2\mu F} + \frac{1}{7\mu F} \quad \text{(237)}
\]

\[
\frac{1}{C_B} = 0.643 \frac{1}{\mu F} \quad \text{(238)}
\]

\[
C_B = 1.555\mu F \quad \text{(239)}
\]

Now finally, we combine \( C_B \) with \( C_4 \) in order to make \( C_C \), the equivalent capacitor to the whole network:  

31
\[ C_C = C_A + C_B \quad (240) \]
\[ C_A = 6\mu F + 1.555\mu F = 7.555\mu F \quad (241) \]

(b) Find the charge on each capacitor.

This is very similar to the previous problem, except that we use \( V = \frac{Q}{C} \) instead of \( V = IR \). Also instead of calculating the dissipated power (capacitors don’t dissipate power, they store energy), we use the formula \( U = \frac{1}{2}QV \). Here is what we know initially:

<table>
<thead>
<tr>
<th></th>
<th>C (( \mu F ))</th>
<th>V (Volts)</th>
<th>Q (( \mu C ))</th>
<th>U (Joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_4</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_A</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_B</td>
<td>1.555</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_C</td>
<td>7.555</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OK; 2 out of 3 on the bottom line and we we fill in:

<table>
<thead>
<tr>
<th></th>
<th>C (( \mu F ))</th>
<th>V (Volts)</th>
<th>Q (( \mu C ))</th>
<th>U (( \mu \text{Joules} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_4</td>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_A</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_B</td>
<td>1.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
<tr>
<td>C_C</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>

OK, now we know that since \( C_C \) is made from the parallel combination of \( C_A \) and \( C_B \) that all three of these share the same voltage:

<table>
<thead>
<tr>
<th></th>
<th>C (( \mu F ))</th>
<th>V (Volts)</th>
<th>Q (( \mu C ))</th>
<th>U (( \mu \text{Joules} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_4</td>
<td>6</td>
<td>10</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td>C_A</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_B</td>
<td>1.555</td>
<td>10</td>
<td>15.55</td>
<td>77.75</td>
</tr>
<tr>
<td>C_C</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>

OK, 2 out of 3 for \( C_A \) and \( C_B \), so we finish out the lines as:

<table>
<thead>
<tr>
<th></th>
<th>C (( \mu F ))</th>
<th>V (Volts)</th>
<th>Q (( \mu C ))</th>
<th>U (( \mu \text{Joules} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_4</td>
<td>6</td>
<td>10</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td>C_A</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_B</td>
<td>1.555</td>
<td>10</td>
<td>15.55</td>
<td>77.75</td>
</tr>
<tr>
<td>C_C</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>
Now we notice that $C_B$ is made from $C_A$ and $C_1$ in series. Thus, all three of these have the same charge:

<table>
<thead>
<tr>
<th></th>
<th>$C$ ($\mu F$)</th>
<th>$V$ (Volts)</th>
<th>$Q$ ($\mu C$)</th>
<th>$U$ ($\mu Joul es$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>2</td>
<td>15.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>6</td>
<td>10</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td>$C_A$</td>
<td>7</td>
<td>15.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_B$</td>
<td>1.555</td>
<td>10</td>
<td>15.55</td>
<td>77.75</td>
</tr>
<tr>
<td>$C_C$</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>

Yup, 2 out of 3 for $C_1$ and $C_A$, so:

<table>
<thead>
<tr>
<th></th>
<th>$C$ ($\mu F$)</th>
<th>$V$ (Volts)</th>
<th>$Q$ ($\mu C$)</th>
<th>$U$ ($\mu Joul es$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>2</td>
<td>7.775</td>
<td>15.55</td>
<td>60.45</td>
</tr>
<tr>
<td>$C_2$</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td>6</td>
<td>10</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td>$C_A$</td>
<td>7</td>
<td>2.221</td>
<td>15.55</td>
<td>17.27</td>
</tr>
<tr>
<td>$C_B$</td>
<td>1.555</td>
<td>10</td>
<td>15.55</td>
<td>77.75</td>
</tr>
<tr>
<td>$C_C$</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>

Finally, $C_2$, $C_3$, and $C_A$ all have the same voltage. Thus, we find:

<table>
<thead>
<tr>
<th></th>
<th>$C$ ($\mu F$)</th>
<th>$V$ (Volts)</th>
<th>$Q$ ($\mu C$)</th>
<th>$U$ ($\mu Joul es$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>2</td>
<td>7.775</td>
<td>15.55</td>
<td>60.45</td>
</tr>
<tr>
<td>$C_2$</td>
<td>3</td>
<td>2.221</td>
<td>6.663</td>
<td>7.399</td>
</tr>
<tr>
<td>$C_3$</td>
<td>4</td>
<td>2.221</td>
<td>8.884</td>
<td>9.866</td>
</tr>
<tr>
<td>$C_4$</td>
<td>6</td>
<td>10</td>
<td>60</td>
<td>300</td>
</tr>
<tr>
<td>$C_A$</td>
<td>7</td>
<td>2.221</td>
<td>15.55</td>
<td>17.27</td>
</tr>
<tr>
<td>$C_B$</td>
<td>1.555</td>
<td>10</td>
<td>15.55</td>
<td>77.75</td>
</tr>
<tr>
<td>$C_C$</td>
<td>7.555</td>
<td>10</td>
<td>75.55</td>
<td>377.75</td>
</tr>
</tbody>
</table>

(c) Find the voltage across each capacitor.
   in table

(d) Find the energy stored in each capacitor.
   in table
14. Shown in the figure below is an electrical circuit. The values of the components are as follows: \( R_1 = 1k\Omega \), \( R_2 = 2k\Omega \), \( R_3 = 3k\Omega \), \( V_1 = 10V \), \( V_2 = 20V \).

(a) Write the Kirchhoff voltage law for the loop containing \( V_1, R_1 \), and \( R_2 \).

I will assume the following three currents:

i. \( I_1 \) flows to the right through \( R_1 \).

ii. \( I_2 \) flows down through \( R_2 \).

iii. \( I_3 \) flows up through \( R_3 \).

In this case, we can solve for the voltage equation:

\[
0 = V_1 - R_1 I_1 - R_2 I_2 \quad (242)
\]

\[
0 = 10 - 1000I_1 - 2000I_2 \quad (243)
\]

\[
10 = 1000I_1 + 2000I_2 \quad (244)
\]

(b) Write the Kirchhoff voltage law for the loop containing \( V_2, R_3 \), and \( R_2 \).

\[
0 = V_2 - R_3 I_3 - R_2 I_2 \quad (245)
\]

\[
0 = 20 - 3000I_3 - 2000I_2 \quad (246)
\]

\[
20 = 3000I_3 + 2000I_2 \quad (247)
\]

(c) Write the Kirchhoff current law.

Looking at the top junction, we find that the currents \( I_1 \) and \( I_3 \) flow in while \( I_2 \) flows out. In this case,

\[
I_1 + I_3 = I_2 \quad (248)
\]

(d) Find the current through \( R_1 \).

The best first step is **always** to put the current equation into the voltage equation. In this case:

\[
10 = 1000I_1 + 2000I_2 \quad (249)
\]

\[
10 = 1000I_1 + 2000(I_1 + I_3) \quad (250)
\]

\[
10 = 3000I_1 + 2000I_3 \quad (251)
\]

\[
20 = 3000I_3 + 2000I_2 \quad (252)
\]

\[
20 = 3000I_3 + 2000(I_1 + I_3) \quad (253)
\]

\[
20 = 5000I_3 + 2000I_1 \quad (254)
\]
OK, to eliminate $I_3$, let’s multiply the first equation by 5 and the second equation by -2. We can then add them:

\[
\begin{align*}
50 &= 15000I_1 + 10000I_3 \\
-40 &= -10000I_3 - 4000I_1 \\
10 &= 11000I_1 \\
I_1 &= 0.000909 = 0.909mA
\end{align*}
\] (255) (256) (257) (258)

(e) Find the current through $R_2$.
Now that we know $I_1$, we can plug this into a previous equation containing $I_1$ and $I_2$:

\[
\begin{align*}
10 &= 1000I_1 + 2000I_2 \\
10 &= 0.909 + 2000I_2 \\
9.091 &= 2000I_2 \\
I_2 &= 0.0045455 = 4.5455mA
\end{align*}
\] (259) (260) (261) (262)

(f) Find the current through $R_3$.
OK, let’s put the $I_1$ and $I_2$ into the current equation:

\[
\begin{align*}
I_1 + I_3 &= I_2 \\
0.909mA + I_3 &= 4.5455mA \\
I_3 &= 3.6365mA
\end{align*}
\] (263) (264) (265)

(g) Find the power dissipated by each resistor.
OK, we can calculate power any of the following ways: $P = VI = I^2R = \frac{V^2}{R}$. Presently we know $R$ and $I$ for every resistor. So, this makes for the simplest calculation:

<table>
<thead>
<tr>
<th>R (Ω)</th>
<th>I (A)</th>
<th>P (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>1000</td>
<td>0.000909</td>
</tr>
<tr>
<td>$R_2$</td>
<td>2000</td>
<td>0.0045455</td>
</tr>
<tr>
<td>$R_3$</td>
<td>3000</td>
<td>0.0036365</td>
</tr>
</tbody>
</table>

36
15. Shown in the figure below is a loop of current placed into a uniform magnetic field. The field is exactly vertical. The plane of the loop, is tipped 15 degrees off the horizontal. The radius of the loop is 20cm and it carried a current of 0.75 Amperes. The current would appear to flow counterclockwise if you viewed the loop from above.

(a) Calculate the magnetic moment of the loop.
This is quite easy:

\[ \mu = IA \]  
\[ \mu = I\pi r^2 \]  
\[ \mu = 0.75 \times \pi \times 0.20^2 \]  
\[ \mu = 0.0942 \text{Amp} \cdot \text{m}^2 \]  

(b) Calculate the torque on the loop.
The torque comes from the cross product:

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]  
\[ |\vec{\tau}| = |\vec{\mu}| |\vec{B}| \sin \theta_{\mu,B} \]  
\[ |\vec{\tau}| = 0.09420.55 \sin 15^\circ \]  
\[ |\vec{\tau}| = 0.0134 \text{N} \cdot \text{m} \]  

(c) Calculate the potential energy resulting from having the loop oriented as shown.
The energy comes from the dot product:

\[ U = -\vec{\mu} \cdot \vec{B} \]  
\[ U = -|\vec{\mu}| |\vec{B}| \cos \theta_{\mu,B} \]  
\[ U = -0.09420.55 \cos 15^\circ \]  
\[ U = -0.500 \text{Joules} \]
16. The circuit below begins with the switch in the open position and the capacitor carrying zero charge. The battery has $V_B = 10V$, the resistor has $R = 5k\Omega$, and the capacitor has $C = 12\mu F$. At time=0, the switch in the circuit is closed.

(a) Determine the time constant of this circuit.
   \[ \tau = RC = 5k\Omega \times 12\mu F = 60\text{msec} \]

(b) Draw sketches of the time dependence of each of the following:
   i. The voltage on the capacitor as a function of time, $V_C(t)$.

   ![Graph of $V_C(t)$]

   ii. The charge on the capacitor as a function of time, $Q_C(t)$.

   ![Graph of $Q_C(t)$]

   iii. The voltage on the resistor as a function of time, $V_R(t)$.

   ![Graph of $V_R(t)$]
iv. The current through the resistor as a function of time.

To be counted for full credit the vertical axis of each sketch should be labelled with a numerical value indicating either the initial or asymptotic value of the quantity plotted.

(c) Write an equation for each of the following:

i. The voltage on the capacitor as a function of time, $V_C(t)$.
   The voltage on the capacitor is a saturating exponential whose eventual voltage is the full $10 \text{ V}$ of the battery. Thus, $V_C(t) = 10V \left(1 - e^{-\frac{t}{\tau_{\text{final}}}}\right)$

ii. The charge on the capacitor as a function of time, $Q_C(t)$.
   Since the asymptotic voltage on the capacitor is $10 \text{ V}$, and $V = \frac{Q}{C}$ or $Q = CV$, the asymptotic charge on the capacitor is $Q_{\text{final}} = 120\mu C$. $Q_C(t) = 120\mu C \left(1 - e^{-\frac{t}{\tau_{\text{final}}}}\right)$

iii. The voltage on the resistor as a function of time, $V_R(t)$.
   The resistor takes up the balance of the battery voltage that is not across the capacitor. This is a decaying exponential: $V_R(t) = 10Ve^{-\frac{t}{\tau_{\text{final}}}}$

iv. The current through the resistor as a function of time.
   Well, $V = IR$ implies that $I = \frac{V}{R}$. Since that starting voltage on the resistor is $10 \text{ V}$, the starting $I = \frac{10V}{60\text{msec}} = 2mA$. $I_R(t) = 2mAe^{-\frac{t}{\tau_{\text{final}}}}$

(d) At what time does the voltage on the capacitor reach $3 \text{ V}$?
   Using the capacitor voltage formula, we plug in $3 \text{ V}$ for the voltage in the $V_C(t)$ formula. Then we have:

\[ 3V = 10V \left(1 - e^{-\frac{t}{\tau_{\text{final}}}}\right) \]  \hspace{1cm} (278)
\[ 0.3 = \left(1 - e^{-\frac{t}{\tau_{\text{final}}}}\right) \]  \hspace{1cm} (279)
\[ 0.7 = e^{-\frac{t}{\tau_{\text{final}}}} \]  \hspace{1cm} (280)
\[ \ln 0.7 = -\frac{t}{60\text{msec}} \]  \hspace{1cm} (281)
\[ t = -60\text{ms} \times \ln 0.7 = 21.4\text{ms} \] (282)