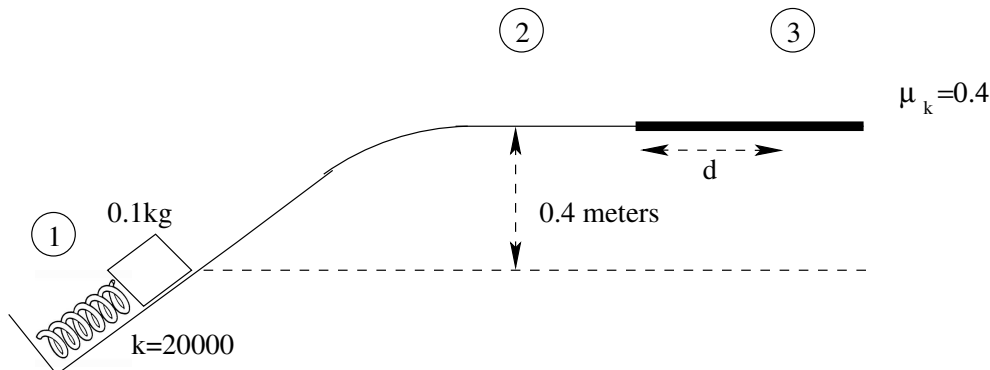


Physics 121
Midterm Examination #2
November 7, 2006

SOLUTIONS

1. Shown in the figure below is a mass pressed against a spring. The spring has constant $k = 20000 \frac{N}{m}$ and at the moment shown in the figure is compressed by $x = 0.01 m$. When released, the mass slides up a frictionless hill. The mass later comes to rest after entering a region with coefficient of friction $\mu_k = 0.4$.



- (a) (10 pts) Find the velocity of the small mass at location 2 (top of the hill).

$$0 + 0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + 0 \quad (1)$$

$$\frac{1}{2}20000(0.01)^2 = \frac{1}{2}0.1v_2^2 + 0.1(9.8)0.4 \quad (2)$$

$$1 = \frac{1}{2}0.1v_2^2 + 0.392 \quad (3)$$

$$1 = \frac{1}{2}0.1v_2^2 + 0.392 \quad (4)$$

$$0.608 = \frac{1}{2}0.1v_2^2 \quad (5)$$

$$v_2^2 = 12.16 \quad (6)$$

$$v_2 = 3.49 \frac{m}{s} \quad (7)$$

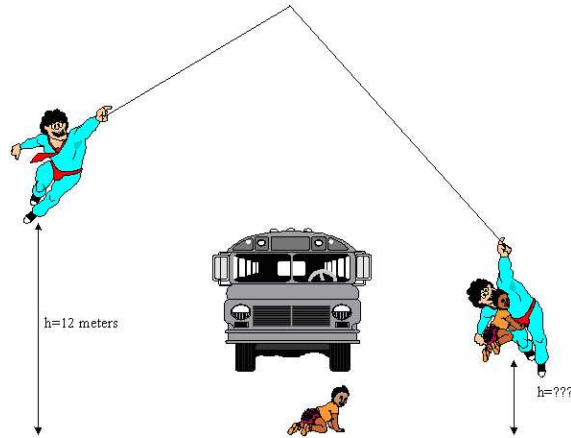
- (b) (10 pts) The mass comes to rest at location 3 (a distance d after entering the frictional area). Find d .

$$-\mu_k mgd = mgy_2 - \frac{1}{2}kx_1^2 \quad (8)$$

$$-\mu_k mgd = 0.1(9.8)0.4 - 1 = -.608 \quad (9)$$

$$d = \frac{.608}{\mu_k mg} = \frac{.608}{0.4(0.1)9.8} = 1.55 m \quad (10)$$

2. One day SuperDude ($M = 80 \text{ kg}$) notices an innocent baby ($m = 20 \text{ kg}$) about to be run down by a bus. SuperDude swings down, snatches the baby in his arms and saves the day. Assuming that SuperDude starts from rest at a height of $h = 12 \text{ m}$, calculate all of the following:



- (a) (7 pts) The velocity of SuperDude just before he grabs the child.

$$0 + MgY_1 + 0 = 1/2MV_2^2 + 0 + 0 \quad (11)$$

$$V_2 = \sqrt{2gY_1} = \sqrt{2(9.8)12} = 15.34 \frac{m}{s} \quad (12)$$

- (b) (7 pts) The velocity of SuperDude and the child just after he grabs the child.

$$mv_1 + MV_1 = mv_2 + MV_2 \quad (13)$$

$$v_2 = V_2 \quad (14)$$

$$MV_1 = (m + M)V_2 \quad (15)$$

$$V_2 = \frac{M}{m + M}V_1 = \frac{80}{10}15.34 = 12.27 \frac{m}{s} \quad (16)$$

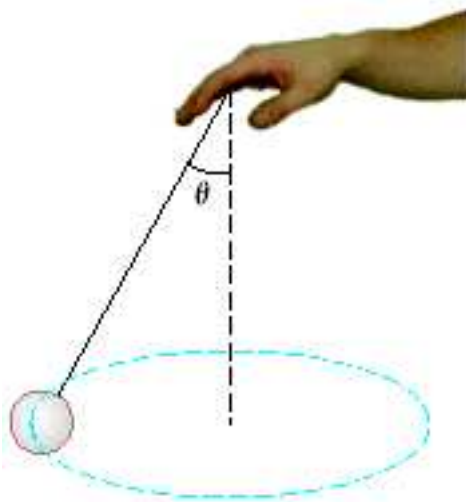
- (c) (6 pts) The height reached by SuperDude and the child.

$$\frac{1}{2}(m + M)V_2^2 + 0 + 0 = 0 + (m + M)gY_3 + 0 \quad (17)$$

$$\frac{1}{2}V_2^2 = gY_3 \quad (18)$$

$$Y_3 = \frac{V_2^2}{2g} = 7.68m \quad (19)$$

3. A 0.60 kg ball that is tied to the end of a 1.2 m light cord is revolved in a horizontal plane with the cord making a 30 degree angle, with the vertical (See Figure).



- (a) (10 pts) Determine the ball's speed.

$$\text{cent :} \quad (20)$$

$$T \sin(\theta) = m \frac{v^2}{R} \quad (21)$$

$$T \sin(\theta) = m \frac{v^2}{L \sin(\theta)} \quad (22)$$

$$T \sin^2(\theta) = m \frac{v^2}{L} \quad (23)$$

$$\text{vert :} \quad (24)$$

$$T \cos(\theta) = mg \quad (25)$$

$$T = \frac{mg}{\cos(\theta)} = \frac{0.6(9.8)}{0.866} = 6.79 \text{ N} \quad (26)$$

$$T \sin^2(\theta) = m \frac{v^2}{L} \quad (27)$$

$$v^2 = \frac{T \sin^2(\theta)L}{m} = \frac{6.79 \sin^2(30)1.2}{0.6} = 3.39 \quad (28)$$

$$v = 1.84 \frac{\text{m}}{\text{s}} \quad (29)$$

- (b) (10 pts) If the cord can withstand a maximum tension of 9.5 N, what is the highest speed at which the ball can move?

$$T \cos(\theta) = mg \quad (30)$$

$$\cos(\theta) = \frac{mg}{T} = \frac{0.6(9.8)}{9.5} = 0.619 \quad (31)$$

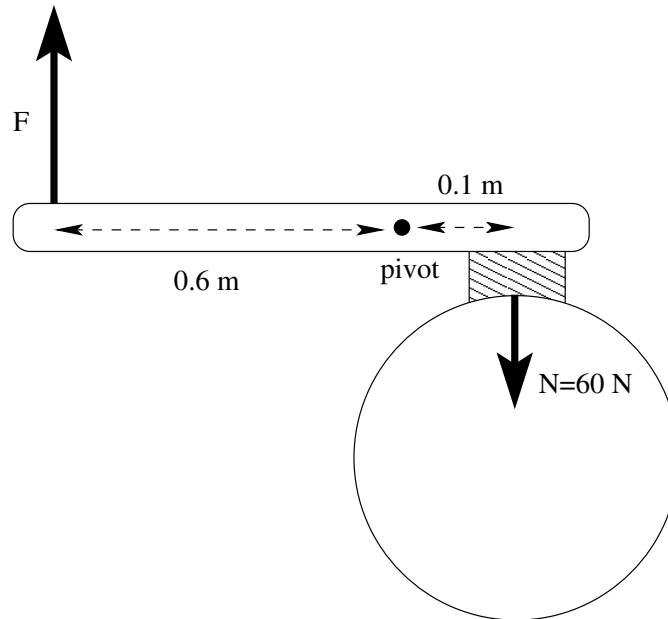
$$\theta = 51.76^\circ \quad (32)$$

$$T \sin^2(\theta) = m \frac{v^2}{L} \quad (33)$$

$$v^2 = \frac{T \sin^2(\theta)L}{m} = \frac{9.5 \sin^2(51.76)1.2}{0.6} = 11.72 \quad (34)$$

$$v = 3.42 \frac{m}{s} \quad (35)$$

4. Shown in the figure below is a simple hand brake that is used to stop a rotating wheel. By applying an upward force F on the end of the lever, you create a downward normal force on the wheel of $N = 50\text{ N}$. The coefficient of friction at the point where the brake touches the wheel is $\mu_k = 0.4$. The wheel has radius $R = 0.4\text{ m}$, mass $m = 3\text{ kg}$, and should be approximated as a solid cylinder.



- (a) (8 pts) Find the torque τ and angular acceleration α of the wheel.

$$\tau = \mu_k N R = 0.4(50)(0.4) = 8.0\text{ Nm} \quad (36)$$

$$\tau = I\alpha \quad (37)$$

$$\alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}mR^2} = \frac{8.0}{0.5(3)0.4^2} = 33.33\frac{\text{rad}}{\text{s}^2} \quad (38)$$

- (b) (8 pts) If the wheel is initially turning at 100 rpm, find the number of turns the wheel makes while coming to a stop.

$$\omega_0 = 100\frac{\text{rev}}{\text{min}} \frac{2\pi\text{ rad}}{1\text{ rev}} \frac{1\text{ min}}{60\text{ sec}} = 10.47\frac{\text{rad}}{\text{s}} \quad (39)$$

$$\theta(\omega = 0) = ? \quad (40)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad (41)$$

$$0 = 10.47^2 + 2(-33.33)(\theta - 0) \quad (42)$$

$$\theta = \frac{10.47^2}{2(33.33)} = 1.64 \text{ rad} \frac{1 \text{ turn}}{2\pi \text{ rad}} = 0.262 \text{ turns} \quad (43)$$

(c) (4 pts) Determine the force F .

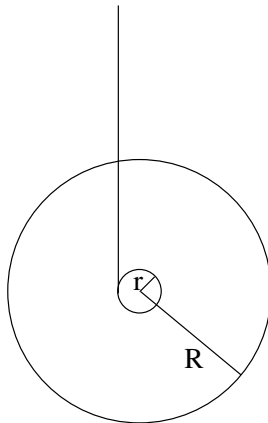
$$Fr_1 - Nr_2 = 0 \quad (44)$$

$$F0.5 - (50)(0.1) \quad (45)$$

$$F = \frac{0.1}{0.5}50 = 10 \text{ N} \quad (46)$$

NOTE: You may ignore the signs of the answers in part a. Do not ignore the signs in part b.

5. Shown in the figure below is a simple Yoyo. The body of the Yoyo has a radius $R = 0.06 \text{ m}$, and the shaft that the string wraps around has radius $r = 0.003 \text{ m}$. Approximate the Yoyo as a solid cylinder of radius R and mass $m = 0.100 \text{ kg}$.



- (a) (10 pts) What is the downward acceleration of the Yoyo?

$$mg - T = ma \quad (47)$$

$$Tr = I\alpha \quad (48)$$

$$I = \frac{1}{2}mR^2 \quad (49)$$

$$\alpha = \frac{a}{r} \quad (50)$$

$$Tr = \frac{1}{2}mR^2 \frac{a}{r} \quad (51)$$

$$T = \frac{1}{2}m \frac{R^2}{r^2} a \quad (52)$$

$$\text{Sum eqns :} \quad (53)$$

$$mg = \left(m + \frac{1}{2}m \frac{R^2}{r^2}\right)a \quad (54)$$

$$g = \left(1 + \frac{1}{2} \frac{R^2}{r^2}\right)a \quad (55)$$

$$a = \frac{g}{\left(1 + \frac{1}{2} \frac{R^2}{r^2}\right)} = 0.049 \frac{m}{s^2} \quad (56)$$

$$(57)$$

- (b) (10 pts) What is the tension in the string?

$$mg - T = ma \quad (58)$$

$$T = mg - ma = m(g - a) = 0.1(9.8 - 0.049) = 0.975 \text{ N} \quad (59)$$