Physics 121
Midterm Examination #2
November 7, 2006

SOLUTIONS
1. Shown in the figure below is a mass pressed against a spring. The spring has constant \( k = 20000 \text{ N/m} \) and at the moment shown in the figure is compressed by \( x = 0.01 \text{ m} \). When released, the mass slides up a frictionless hill. The mass later comes to rest after entering a region with coefficient of friction \( \mu_k = 0.4 \).

(a) (10 pts) Find the velocity of the small mass at location 2 (top of the hill).

\[
0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgx_2 + 0 \quad (1)
\]
\[
\frac{1}{2} \cdot 20000 \cdot (0.01)^2 = \frac{1}{2} \cdot 0.1v_2^2 + 0.1 \cdot (9.8) \cdot 0.4 \quad (2)
\]
\[
1 = \frac{1}{2} \cdot 0.1v_2^2 + 0.392 \quad (3)
\]
\[
1 = \frac{1}{2} \cdot 0.1v_2^2 + 0.392 \quad (4)
\]
\[
0.608 = \frac{1}{2} \cdot 0.1v_2^2 \quad (5)
\]
\[
v_2^2 = 12.16 \quad (6)
\]
\[
v_2 = 3.49 \frac{m}{s} \quad (7)
\]

(b) (10 pts) The mass comes to rest at location 3 (a distance \( d \) after entering the frictional area). Find \( d \).

\[
-\mu_k mgd = mgy_2 - \frac{1}{2}kx_1^2 \quad (8)
\]
\[
-\mu_k mgd = 0.1 \cdot (9.8) \cdot 0.4 - 1 = -0.608 \quad (9)
\]
\[
d = \frac{0.608}{\mu_k mg} = \frac{0.608}{0.4 \cdot (0.1) \cdot 9.8} = 1.55 \text{ m} \quad (10)
\]
2. One day SuperDude \((M = 80\ kg)\) notices an innocent baby \((m = 20\ kg)\) about to be run down by a bus. SuperDude swings down, snatches the baby in his arms and saves the day. Assuming that SuperDude starts from rest at a height of \(h = 12\ m\), calculate all of the following:

(a) (7 pts) The velocity of SuperDude just before he grabs the child.

\[
0 + MgY_1 + 0 = 1/2MV_2^2 + 0 + 0
\]
\[
V_2 = \sqrt{2gY_1} = \sqrt{2(9.8)(12)} = 15.34 \frac{m}{s} \tag{12}
\]

(b) (7 pts) The velocity of SuperDude and the child just after he grabs the child.

\[
mv_1 + MV_1 = mv_2 + MV_2
\]
\[
v_2 = V_2 \tag{14}
\]
\[
MV_1 = (m + M)V_2
\]
\[
V_2 = \frac{M}{m + M} V_1 = \frac{80}{10} \times 15.34 = 12.27 \frac{m}{s} \tag{16}
\]

(c) (6 pts) The height reached by SuperDude and the child.

\[
\frac{1}{2}(m + M)V_2^2 + 0 + 0 = 0 + (m + M)gY_3 + 0
\]
\[
\frac{1}{2}V_2^2 = gY_3 \tag{18}
\]
\[
Y_3 = \frac{V_2^2}{2g} = 7.68m \tag{19}
\]
3. A 0.60 kg ball that is tied to the end of a 1.2 m light cord is revolved in a horizontal plane with the cord making a 30 degree angle, with the vertical (See Figure).

(a) (10 pts) Determine the ball’s speed.

\[
\begin{align*}
\text{cent:} & \\
T \sin(\theta) &= m \frac{v^2}{R} \\
T \sin(\theta) &= m \frac{v^2}{L \sin(\theta)} \\
T \sin^2(\theta) &= m \frac{v^2}{L} \\
\text{vert:} & \\
T \cos(\theta) &= mg
\end{align*}
\]

\[
T = \frac{mg}{\cos(\theta)} = \frac{0.6(9.8)}{0.866} = 6.79 \text{ N}
\]

\[
T \sin^2(\theta) = m \frac{v^2}{L}
\]

\[
v^2 = \frac{T \sin^2(\theta) L}{m} = \frac{6.79 \sin^2(30) 1.2}{0.6} = 3.39
\]

\[
v = 1.84 \frac{m}{s}
\]
(b) (10 pts) If the cord can withstand a maximum tension of 9.5 N, what is the highest speed at which the ball can move?

\[ T \cos(\theta) = mg \quad (30) \]

\[ \cos(\theta) = \frac{mg}{T} = \frac{0.6(9.8)}{9.5} = 0.619 \quad (31) \]

\[ \theta = 51.76^\circ \quad (32) \]

\[ T \sin^2(\theta) = \frac{mv^2}{L} \quad (33) \]

\[ v^2 = \frac{T \sin^2(\theta)L}{m} = \frac{9.5 \sin^2(51.76) \cdot 1.2}{0.6} = 11.72 \quad (34) \]

\[ v = 3.42 \frac{m}{s} \quad (35) \]
4. Shown in the figure below is a simple hand brake that is used to stop a rotating wheel. By applying an upward force $F$ on the end of the lever, you create a downward normal force on the wheel of $N = 50$ N. The coefficient of friction at the point where the brake touches the wheel is $\mu_k = 0.4$. The wheel has radius $R = 0.4$ m, mass $m = 3$ kg, and should be approximated as a solid cylinder.

![Diagram of a rotating wheel with a lever and brake](image)

(a) (8 pts) Find the torque $\tau$ and angular acceleration $\alpha$ of the wheel.

$$
\tau = \mu_k NR = 0.4(50)(0.4) = 8.0 \text{ Nm}
$$

$$
\tau = I\alpha \quad (36)
$$

$$
\alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}mR^2} = \frac{8.0}{0.5(3)0.4^2} = 33.33 \frac{rad}{s^2} \quad (38)
$$

(b) (8 pts) If the wheel is initially turning at 100 rpm, find the number of turns the wheel makes while coming to a stop.

$$
\omega_0 = 100 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 10.47 \frac{\text{rad}}{s} \quad (39)
$$

$$\theta(\omega = 0) = ? \quad (40)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad (41)$$

$$0 = 10.47^2 + 2(-33.33)(\theta - 0) \quad (42)$$
\[ \theta = \frac{10.47^2}{2(33.33)} = 1.64 \text{ rad} \frac{1 \text{ turn}}{2\pi \text{ rad}} = 0.262 \text{ turns} \quad (43) \]

(c) (4 pts) Determine the force F.

\[ F r_1 - N r_2 = 0 \quad (44) \]
\[ F 0.5 - (50)(0.1) \quad (45) \]
\[ F = \frac{0.1}{0.5} = 10 \text{ N} \quad (46) \]

**NOTE:** You may ignore the signs of the answers in part a. Do not ignore the signs in part b.
5. Shown in the figure below is a simple Yoyo. The body of the Yoyo has a radius \( R = 0.06 \, \text{m} \), and the shaft that the string wraps around has radius \( r = 0.003 \, \text{m} \). Approximate the Yoyo as a solid cylinder of radius \( R \) and mass \( m = 0.100 \, \text{kg} \).

![Diagram of Yoyo](image)

(a) (10 pts) What is the downward acceleration of the Yoyo?

\[
m g - T = ma \tag{47}
\]

\[
Tr = I \alpha \tag{48}
\]

\[
I = \frac{1}{2} m R^2 \tag{49}
\]

\[
a = \frac{a}{r} \tag{50}
\]

\[
Tr = \frac{1}{2} m R^2 \frac{a}{r} \tag{51}
\]

\[
T = \frac{1}{2} m \frac{R^2}{r^2} a \tag{52}
\]

Sum eqns:

\[
m g = (m + \frac{1}{2} m \frac{R^2}{r^2}) a \tag{53}
\]

\[
g = (1 + \frac{1}{2} \frac{R^2}{r^2}) a \tag{54}
\]

\[
a = \frac{g}{(1 + \frac{R^2}{r^2})} = 0.049 \frac{m}{s^2} \tag{55}
\]

(b) (10 pts) What is the tension in the string?

\[
m g - T = ma \tag{56}
\]
\[ T = mg - ma = m(g - a) = 0.1(9.8 - 0.049) = 0.975 \text{ N} \] (59)