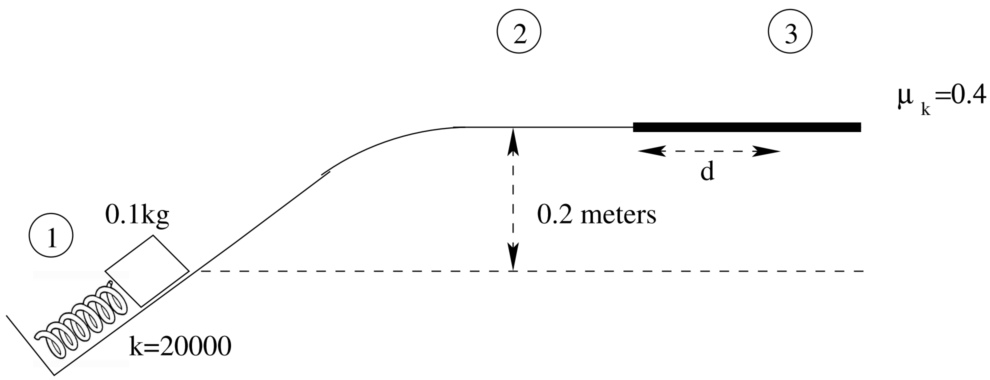
Physics 121
Midterm Examination #2
November 7, 2006

SOLUTIONS

1. Shown in the figure below is a mass pressed against a spring. The spring has constant $k = 20000\ \frac{N}{m}$ and at the moment shown in the figure is compressed by $x = 0.01\,m$. When released, the mass slides up a frictionless hill. The mass later comes to rest after entering a region with coefficient of friction $\mu_k = 0.4$.



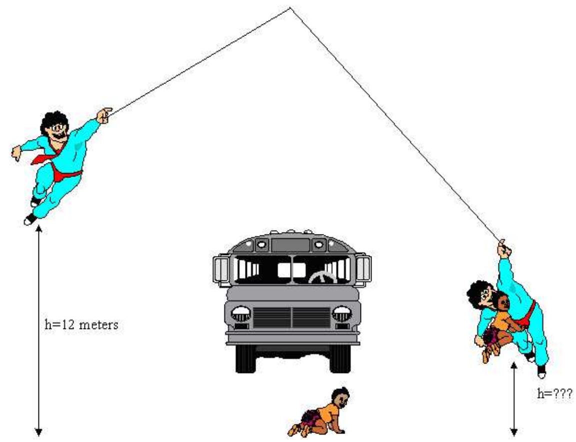
(a) (10 pts) Find the velocity of the small mass at location 2 (top of the hill).

$$0 + 0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + 0 \quad (1)$$
$$\frac{1}{2}20000(0.01)^2 = \frac{1}{2}0.1v_2^2 + 0.1(9.8)0.2 \quad (2)$$
$$1 = \frac{1}{2}0.1v_2^2 + 0.196 \quad (3)$$
$$1 = \frac{1}{2}0.1v_2^2 + 0.196 \quad (4)$$
$$0.804 = \frac{1}{2}0.1v_2^2 \quad (5)$$
$$v_2^2 = 16.08 \quad (6)$$
$$v_2 = 4.01\frac{m}{s} \quad (7)$$

(b) (10 pts) The mass comes to rest at location 3 (a distance d after entering the frictional area). Find d.

$$-\mu_k mgd = mgy_2 - \frac{1}{2}kx_1^2 \quad (8)$$
$$-\mu_k mgd = 0.1(9.8)0.2 - 1 = -.804 \quad (9)$$
$$d = \frac{.804}{\mu_k mg} = \frac{.804}{0.4(0.1)9.8} = 2.05\ m \quad (10)$$

2. One day SuperDude ($M = 100\ kg$) notices an innocent baby ($m = 15\ kg$) about to be run down by a bus. SuperDude swings down, snatches the baby in his arms and saves the day. Assuming that SuperDude starts from rest at a height of $h = 12\ m$, calculate all of the following:



(a) (7 pts) The velocity of SuperDude just before he grabs the child.

$$0 + MgY_1 + 0 = 1/2MV_2^2 + 0 + 0 \tag{11}$$
$$V_2 = \sqrt{2gY_1} = \sqrt{2(9.8)12} = 15.34\frac{m}{s} \tag{12}$$

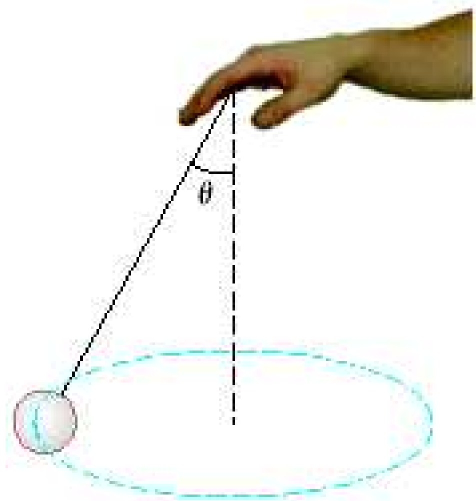
(b) (7 pts) The velocity of SuperDude and the child just after he grabs the child.

$$mv_1 + MV_1 = mv_2 + MV_2 \tag{13}$$
$$v_2 = V_2 \tag{14}$$
$$MV_1 = (m + M)V_2 \tag{15}$$
$$V_2 = \frac{M}{m + M}V_1 = \frac{100}{115}15.34 = 13.34\frac{m}{s} \tag{16}$$

(c) (6 pts) The height reached by SuperDude and the child.

$$\frac{1}{2}(m + M)V_2^2 + 0 + 0 = 0 + (m + M)gY_3 + 0 \tag{17}$$
$$\frac{1}{2}V_2^2 = gY_3 \tag{18}$$
$$Y_3 = \frac{V_2^2}{2g} = 9.07m \tag{19}$$

3. A 0.50 kg ball that is tied to the end of a 1.3 m light cord is revolved in a horizontal plane with the cord making a 30 degree angle, with the vertical (See Figure).



(a) (10 pts) Determine the ball's speed.

$$cent: \tag{20}$$

$$T\sin(\theta) = m\frac{v^2}{R} \tag{21}$$

$$T\sin(\theta) = m\frac{v^2}{L\sin(\theta)} \tag{22}$$

$$T\sin^2(\theta) = m\frac{v^2}{L} \tag{23}$$

$$vert: \tag{24}$$

$$T\cos(\theta) = mg \tag{25}$$

$$T = \frac{mg}{\cos(\theta)} = \frac{0.5(9.8)}{0.866} = 5.66\ N \tag{26}$$

$$T\sin^2(\theta) = m\frac{v^2}{L} \tag{27}$$

$$v^2 = \frac{T\sin^2(\theta)L}{m} = \frac{5.66\sin^2(30)1.3}{0.5} = 3.68 \tag{28}$$

$$v = 1.92\frac{m}{s} \tag{29}$$

(b) (10 pts) If the cord can withstand a maximum tension of 9.9 N, what is the highest speed at which the ball can move?

$$T\cos(\theta) = mg \tag{30}$$

$$\cos(\theta) = \frac{mg}{T} = \frac{0.5(9.8)}{9.9} = 0.4949 \tag{31}$$

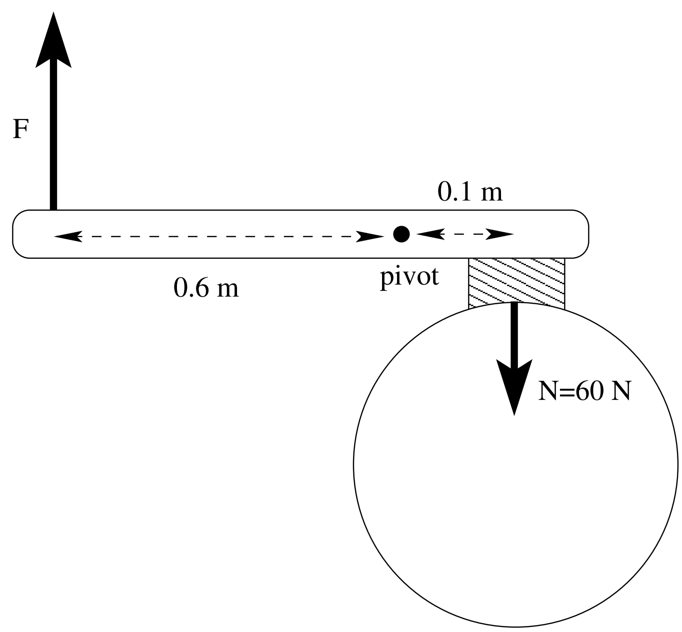
$$\theta = 60.33^o \tag{32}$$

$$T\sin^2(\theta) = m\frac{v^2}{L} \tag{33}$$

$$v^2 = \frac{T\sin^2(\theta)L}{m} = \frac{9.9\sin^2(60.33)1.3}{0.5} = 19.43 \tag{34}$$

$$v = 4.41\frac{m}{s} \tag{35}$$

4. Shown in the figure below is a simple hand brake that is used to stop a rotating wheel. By applying an upward force F on the end of the lever, you create a downward normal force on the wheel of $N = 60\ N$. The coefficient of friction at the point where the brake touches the wheel is $\mu_k = 0.4$. The wheel has radius $R = 0.4\ m$, mass $m = 3\ kg$, and should be approximated as a solid cylinder.



(a) (8 pts) Find the torque τ and angular acceleration α of the wheel.

$$\tau = \mu_k N R = 0.4(60)(0.4) = 9.6\ Nm \quad (36)$$
$$\tau = I\alpha \quad (37)$$
$$\alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}mR^2} = \frac{9.6}{0.5(3)0.4^2} = 40\ \frac{rad}{s^2} \quad (38)$$

(b) (8 pts) If the wheel is initially turning at 100 rpm, find the number of turns the wheel makes while coming to a stop.

$$\omega_0 = 100\frac{rev}{min}\frac{2\pi\ rad}{1\ rev}\frac{1\ min}{60\ sec} = 10.47\frac{rad}{s} \quad (39)$$
$$\theta(\omega = 0) = ? \quad (40)$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad (41)$$
$$0 = 10.47^2 + 2(-40)(\theta - 0) \quad (42)$$

$$\theta = \frac{10.47^2}{2(40)} = 1.37\ rad \frac{1\ turn}{2\pi\ rad} = 0.218 turns \tag{43}$$

(c) (4 pts) Determine the force F.

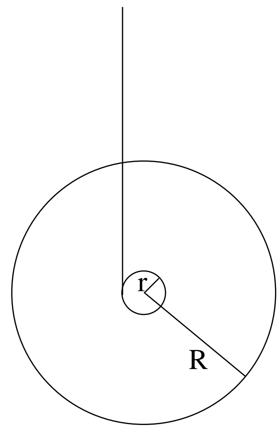
$$Fr_1 - Nr_2 = 0 \tag{44}$$

$$F0.6 - (60)(0.1) \tag{45}$$

$$F = \frac{0.1}{0.6} 60 = 10\ N \tag{46}$$

NOTE: You may ignore the signs of the answers in part a. Do not ignore the signs in part b.

5. Shown in the figure below is a simple Yoyo. The body of the Yoyo has a radius $R = 0.04\ m$, and the shaft that the string wraps around has radius $r = 0.002\ m$. Approximate the Yoyo as a solid cylinder of radius R and mass $m = 0.100\ kg$.



(a) (10 pts) What is the downward acceleration of the Yoyo?

$$mg - T = ma \quad (47)$$
$$Tr = I\alpha \quad (48)$$
$$I = \frac{1}{2}mR^2 \quad (49)$$
$$\alpha = \frac{a}{r} \quad (50)$$
$$Tr = \frac{1}{2}mR^2\frac{a}{r} \quad (51)$$
$$T = \frac{1}{2}m\frac{R^2}{r^2}a \quad (52)$$
$$Sum\ eqns: \quad (53)$$
$$mg = (m + \frac{1}{2}m\frac{R^2}{r^2})a \quad (54)$$
$$g = (1 + \frac{1}{2}\frac{R^2}{r^2})a \quad (55)$$
$$a = \frac{g}{(1 + \frac{1}{2}\frac{R^2}{r^2})} = 0.049\frac{m}{s^2} \quad (56)$$
$$(57)$$

(b) (10 pts) What is the tension in the string?

$$mg - T = ma \quad (58)$$

$$T = mg - ma = m(g - a) = 0.1(9.8 - 0.049) = 0.975\ N \tag{59}$$