Physics 121
Midterm Examination #2
November 7, 2006

SOLUTIONS
1. Shown in the figure below is a mass pressed against a spring. The spring has constant \( k = 20000 \, \text{N/m} \) and at the moment shown in the figure is compressed by \( x = 0.01 \, \text{m} \). When released, the mass slides up a frictionless hill. The mass later comes to rest after entering a region with coefficient of friction \( \mu_k = 0.4 \).

(a) (10 pts) Find the velocity of the small mass at location 2 (top of the hill).

\[
0 + 0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + 0
\]  
(1)

\[
\frac{1}{2} \times 20000 \times (0.01)^2 = \frac{1}{2} \times 0.1v_0^2 + 0.1 \times (9.8) \times 0.2
\]  
(2)

\[
1 = \frac{1}{2} \times 0.1v_0^2 + 0.196
\]  
(3)

\[
1 = \frac{1}{2} \times 0.1v_0^2 + 0.196
\]  
(4)

\[
0.804 = \frac{1}{2} \times 0.1v_0^2
\]  
(5)

\[
v_0^2 = 16.08
\]  
(6)

\[
v_2 = 4.01 \frac{m}{s}
\]  
(7)

(b) (10 pts) The mass comes to rest at location 3 (a distance \( d \) after entering the frictional area). Find \( d \).

\[
-\mu_k mgd = mgy_2 - \frac{1}{2}kx_1^2
\]  
(8)

\[
-\mu_k mgd = 0.1 \times (9.8) \times 0.2 - 1 = -0.804
\]  
(9)

\[
d = \frac{0.804}{\mu_k mg} = \frac{0.804}{0.4 \times (0.1) \times 9.8} = 2.05 \, \text{m}
\]  
(10)
2. One day SuperDude \((M = 100 \text{ kg})\) notices an innocent baby \((m = 15 \text{ kg})\) about to be run down by a bus. SuperDude swings down, snatches the baby in his arms and saves the day. Assuming that SuperDude starts from rest at a height of \(h = 12 \text{ m}\), calculate all of the following:

(a) (7 pts) The velocity of SuperDude just before he grabs the child.

\[
0 + MgY_1 + 0 = \frac{1}{2}MV_2^2 + 0 + 0 \tag{11}
\]

\[
V_2 = \sqrt{2gY_1} = \sqrt{2(9.8)(12)} = 15.34 \frac{m}{s} \tag{12}
\]

(b) (7 pts) The velocity of SuperDude and the child just after he grabs the child.

\[
mv_1 + MV_1 = mv_2 + MV_2 \tag{13}
\]

\[
v_2 = V_2 \tag{14}
\]

\[
MV_1 = (m + M)V_2 \tag{15}
\]

\[
V_2 = \frac{M}{m + M}V_1 = \frac{100}{115}15.34 = 13.34 \frac{m}{s} \tag{16}
\]

(c) (6 pts) The height reached by SuperDude and the child.

\[
\frac{1}{2}(m + M)V_2^2 + 0 + 0 = 0 + (m + M)gY_3 + 0 \tag{17}
\]

\[
\frac{1}{2}V_2^2 = gY_3 \tag{18}
\]

\[
Y_3 = \frac{V_2^2}{2g} = 9.07 \text{ m} \tag{19}
\]
3. A 0.50 kg ball that is tied to the end of a 1.3 m light cord is revolved in a horizontal plane with the cord making a 30 degree angle, with the vertical (See Figure).

(a) (10 pts) Determine the ball's speed.

\[ T \sin(\theta) = m \frac{v^2}{L} \]  
\[ T \sin(\theta) = m \frac{v^2}{L \sin(\theta)} \]  
\[ T \sin^2(\theta) = m \frac{v^2}{L} \]  
\[ T \cos(\theta) = mg \]  

\[ T = \frac{mg}{\cos(\theta)} = \frac{0.5(9.8)}{0.866} = 5.66 \text{ N} \]  
\[ T \sin^2(\theta) = m \frac{v^2}{L} \]  
\[ v^2 = \frac{T \sin^2(\theta) L}{m} = \frac{5.66 \sin^2(30) \times 1.3}{0.5} = 3.68 \]  
\[ v = 1.92 \frac{m}{s} \]
(b) (10 pts) If the cord can withstand a maximum tension of 9.9 N, what is the highest speed at which the ball can move?

\[ T \cos(\theta) = mg \quad (30) \]

\[ \cos(\theta) = \frac{mg}{T} = \frac{0.5(9.8)}{9.9} = 0.4949 \quad (31) \]

\[ \theta = 60.33^\circ \quad (32) \]

\[ T \sin^2(\theta) = m\frac{v^2}{L} \quad (33) \]

\[ v^2 = \frac{T \sin^2(\theta)L}{m} = \frac{9.9 \sin^2(60.33^\circ)1.3}{0.5} = 19.43 \quad (34) \]

\[ v = 4.41 \frac{m}{s} \quad (35) \]
4. Shown in the figure below is a simple hand brake that is used to stop a rotating wheel. By applying an upward force $F$ on the end of the lever, you create a downward normal force on the wheel of $N = 60 \text{ N}$. The coefficient of friction at the point where the brake touches the wheel is $\mu_k = 0.4$. The wheel has radius $R = 0.4 \text{ m}$, mass $m = 3 \text{ kg}$, and should be approximated as a solid cylinder.

![Diagram of a hand brake and a rotating wheel](image)

(a) (8 pts) Find the torque $\tau$ and angular acceleration $\alpha$ of the wheel.

\[
\tau = \mu_k NR = 0.4(60)(0.4) = 9.6 \text{ Nm} \tag{36}
\]

\[
\alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}mR^2} = \frac{9.6}{0.5(3)0.4^2} = 40 \frac{\text{rad}}{s^2} \tag{38}
\]

(b) (8 pts) If the wheel is initially turning at 100 rpm, find the number of turns the wheel makes while coming to a stop.

\[
\omega_0 = 100 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 10.47 \frac{\text{rad}}{s} \tag{39}
\]

\[
\theta(\omega = 0) = ? \tag{40}
\]

\[
\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \tag{41}
\]

\[
0 = 10.47^2 + 2(-40)(\theta - 0) \tag{42}
\]
\[ \theta = \frac{10.47^2}{2(40)} = 1.37 \text{ rad} \times \frac{1 \text{ turn}}{2\pi \text{ rad}} = 0.218 \text{ turns} \]  

(c) (4 pts) Determine the force \( F \).

\[ Fr_1 - Nr_2 = 0 \]  
\[ F0.6 - (60)(0.1) \]  
\[ F = \frac{0.1}{0.6} = 10 \text{ N} \]

**NOTE:** You may ignore the signs of the answers in part a. Do not ignore the signs in part b.
5. Shown in the figure below is a simple Yoyo. The body of the Yoyo has a radius \( R = 0.04 \text{ m} \), and the shaft that the string wraps around has radius \( r = 0.002 \text{ m} \). Approximate the Yoyo as a solid cylinder of radius \( R \) and mass \( m = 0.100 \text{ kg} \).

![Diagram of Yoyo](image)

(a) (10 pts) What is the downward acceleration of the Yoyo?

\[
mg - T = ma \\
Tr = I\alpha \\
I = \frac{1}{2}mR^2 \\
\alpha = \frac{a}{r} \\
Tr = \frac{1}{2}mR^2 \frac{a}{r} \\
T = \frac{1}{2}m \frac{R^2}{r^2}a \\
\text{Sum eqns:} \\
mg = (m + \frac{1}{2}m \frac{R^2}{r^2})a \\
g = (1 + \frac{1}{2} \frac{R^2}{r^2})a \\
a = \frac{g}{(1 + \frac{1}{2} \frac{R^2}{r^2})} = 0.049 \frac{m}{s^2} \tag{57}
\]

(b) (10 pts) What is the tension in the string?

\[
mg - T = ma \tag{58}
\]
\[ T = mg - ma = m(g - a) = 0.1(9.8 - 0.049) = 0.975 \, N \] (59)