Midterm exam:
(a closed book exam, no notes)

March 11 (Wednesday), 2015
Lecture 6:
2. Nucleon Structure – continued
   - Deep Inelastic Scattering
   - Nucleon Structure Functions
Textbook: Povh et. al, Chapter 7
High energy electron beam

J.I. Friedman, H.W. Kendall, R.E. Taylor

"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"
Deep Inelastic Scattering

Lepton beam

\[ p = (E, \bar{p}) \]

Proton (fixed target)

\[ P = (M, 0) \]

Final state hadrons (invariant mass \( W \))

\[ q = (\nu, \bar{q}) \]

Scattered leptons

\[ p' = (E', \bar{p}') \]

Kinematics

Virtual photon mass (hard scale):

\[ -Q^2 = q^2 = (p - p')^2 = (E - E')^2 - (\bar{p} - \bar{p}')^2 \]

\[ Q^2 \approx 4EE' \sin^2 \frac{\theta}{2} > 0 \]

\[ P \cdot q = M(E - E') = M\nu \]

\[ W^2 = P^{'2} = (P + q)^2 = M^2 + 2P \cdot q + q^2 = M^2 + 2M\nu - Q^2 \]

Elastic scattering:

\[ W = M \Rightarrow 2M\nu - Q^2 = 0 \]

Inelastic scattering:

\[ W > M \Rightarrow 2M\nu - Q^2 > 0 \]

- **Inclusive** scattering (final hadronic state not measured)

\[ e^- + p \rightarrow e^- + X \]
Deep Inelastic Scattering

Production of resonances (excited nucleon states) up to $W \sim 1.8$ GeV DIS (Deep Inelastic Scattering) region above $W \sim 1.8$ GeV (no resonance structure)
Deep Inelastic Scattering

Cross section drops off with $Q^2$ in the resonance region (small $W$)
Deep Inelastic Scattering

\[ e^- + p \rightarrow e^- + X \]

Virtual photon mass (hard scale):

\[ -Q^2 = q^2 = (p - p')^2 \]

\[ Q^2 = 2M\nu + M^2 - W^2 \]

\[ \nu = (E - E') \] - energy transfer from the electron to the proton (energy of the virtual photon)

\[ a \leq \nu \leq b \]

**Question 1:** what is the value of \( a \) and \( b \)?

**Question 2:** why straight lines for fixed \( \theta \)

(\text{answer at home})
Electron-Proton Scattering Cross Section

\[ Q^2 = 2M \nu + M^2 - W^2 \]
Deep Inelastic Scattering

Rosenbluth formula:

\[
\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M^* \cdot \left[ W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2} \right]
\]

- 2 structure functions \( W_{1,2} \) depend on 2 independent variables
- Electric interaction
- Magnetic interaction
Reminder

Elastic Scattering Cross Section:
Dirac particle scattering from spin $\frac{1}{2}$ Finite Size Target

- Spin effects included (electron spin=1/2, finite size target spin=1/2)
- Recoil not neglected

Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2} \right]$$

2 structure functions:
$G_E(Q^2)$ and $G_M(Q^2)$ electric and magnetic form factors

if $\frac{q^2}{4M^2} \ll 1 \Rightarrow q^2 \approx -q^2$

and $G(q^2) \approx G(-q^2)$

$G_E(q^2) \approx G_E(-q^2) = \int e^{iq\cdot\vec{x}} f(\vec{x}) d^3x$

$G_M(q^2) \approx G_M(-q^2) = \int e^{iq\cdot\vec{x}} \mu(\vec{x}) d^3x$

Fourier transforms of the charge and magnetic moment distributions (f-charge density function)
Deep Inelastic Scattering

Rosenbluth formula:
\[
\left( \frac{d\sigma}{d\Omega dE'} \right) = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}}^* \cdot \left[ W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \frac{\theta}{2} \right]
\]

2 structure functions \( W_{1,2} \) depend on 2 independent variables

SLAC-MIT experiment (1969)

Experimental result:
\( W_{1,2} \) - nearly independent of \( Q^2 \)

Unexpected result
Deep Inelastic Scattering

\[
\frac{d\sigma}{d\Omega dE'} = \left( \frac{d\sigma}{d\Omega} \right)^*_{\text{Mott}} \cdot \left[ W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \theta \right]
\]

Rosenbluth formula: \( \frac{d\sigma}{d\Omega dE'} \)

2 structure functions \( W_{1,2} \) depend on 2 independent variables

Experimental result:
\( W_{1,2} \) - nearly independent of \( Q^2 \)

SLAC-MIT experiment (1969)
Deep Inelastic Scattering

Experimental result:
$W_{1,2}$ - nearly independent of $Q^2$

SLAC-MIT experiment (1969)

The Nobel Prize in Physics 1990

The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor “for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics”.
Deep Inelastic Scattering – Bjorken Scaling

Bjorken x:

\[ x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M \cdot q} = \frac{Q^2}{2M \nu} \]

\[ 0 \leq x \leq 1 \]

- Elastic scattering:
  \[ W = M \Rightarrow 2M \nu - Q^2 = 0 \Rightarrow x = 1 \]

- Inelastic scattering:
  \[ W > M \Rightarrow 2M \nu - Q^2 > 0 \Rightarrow 0 < x < 1 \]

Bjorken x: physical interpretation
Deep Inelastic Scattering and the Parton Model

Partons: constituents of the proton

\[ p = (E, \vec{p}) \]
Lepton

\[ p' = (E', \vec{p}') \]
Scattered lepton

\[ \gamma, Z^0 (q) \]
Virtual photon

\[ q_f (xP) \]
Parton (hadrons)

Spectator partons (hadrons)

Proton (fixed target)

Interpretation of DIS off protons:

- Infinite momentum frame (Breit frame) \[ |\vec{P}| \gg m_{\text{parton}}, |\vec{P}_{T\text{parton}}|, M \]
- Proton=free moving partons (longitudinal momenta only)
- Lepton-proton DIS interaction = incoherent sum of elastic scattering processes of virtual photon (or boson Z^0) with individual partons (“impulse approximation”)

\[ \text{add probabilities, not amplitudes} \]

This picture does not work in the low energy kinematic region \( \nu \sim M \)
Deep Inelastic Scattering and the Parton Model

Laboratory frame

Breit frame $q = (0, \vec{q})$

<table>
<thead>
<tr>
<th>Proton</th>
<th>Parton</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>$E$</td>
</tr>
<tr>
<td>3-momentum</td>
<td>$\vec{P}$</td>
</tr>
</tbody>
</table>
Exercise:
Show that Bjorken $x = \xi$ of the proton momentum carried by the hit parton in the Breit frame

$$k = \xi P$$ 4-momentum of the parton
$$k' = k + q$$ 4-momentum of the scattered parton
$$k^2 = k'^2$$

$\xi = ?$
Exercise:
Show that Bjorken $x$ = fraction $\xi$ of the proton momentum carried by the hit parton in the Breit frame

$k = \xi P$ \quad 4-momentum of the parton

$k' = k + q$ \quad 4-momentum of the scattered parton

$k^2 = k'^2$

\[
\Rightarrow \quad k^2 = (k + q)^2 = k^2 + 2k \cdot q + q^2 = k^2 + 2\xi P \cdot q - Q^2
\]

\[
\Rightarrow \quad 2\xi P \cdot q - Q^2 = 0
\]

\[
\Rightarrow \quad \xi = \frac{Q^2}{2P \cdot q} \equiv x
\]

Bjorken $x$ interpretation:
x corresponds to proton momentum fraction carried by the struck parton in the proton infinite momentum frame
Deep Inelastic Scattering – Bjorken Scaling

Bjorken x:

\[ x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M \cdot q} = \frac{Q^2}{2M \nu} \]

\( 0 \leq x \leq 1 \)

- Elastic scattering:
  \( W = M \Rightarrow 2M \nu - Q^2 = 0 \Rightarrow x = 1 \)

- Inelastic scattering:
  \( W > M \Rightarrow 2M \nu - Q^2 > 0 \Rightarrow 0 < x < 1 \)

\[ MW_1(Q^2, \nu) = F_1(Q^2, x) \quad \text{(dimensionless)} \]
\[ \nu W_2(Q^2, \nu) = F_2(Q^2, x) \]

Scaling (Bjorken, 1969):

\[ Q^2 \rightarrow \infty \quad \text{and} \quad \nu \rightarrow \infty \]
\[ x = \frac{Q^2}{2M \nu} \quad \text{fixed} \]

Nucleons have a sub-structure made of point-like constituents.
Point particles cannot be further resolved; their measurement does not depend on wavelength, hence \( Q^2 \).
The Callan-Gross relation

For \( \frac{1}{2} \) Dirac particles:
(elastic scattering)

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Dirac}} \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ 1 + 2\tau \tan^2 \theta \right]
\]

\( \tau = \frac{Q^2}{4M^2} \)

For \( \frac{1}{2} \) Dirac particles:

\[
\left( \frac{d\sigma}{d\Omega dE'} \right) = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2 \frac{\theta}{2} \right]
\]

\( Q^2 \rightarrow \infty \) and \( \nu \rightarrow \infty \)

\[
x = \frac{Q^2}{2M\nu} \text{ fixed}
\]

\( MW_1(Q^2, \nu) \rightarrow F_1(x) \)

\( \nu W_2(Q^2, \nu) \rightarrow F_2(x) \)

For spin \( \frac{1}{2} \) Dirac particle:

\[
2xF_1(x) = F_2(x)
\]

target = point-like spin \( \frac{1}{2} \) particle

target = finite size spin \( \frac{1}{2} \) particle
The point-like constituents of the nucleon have spin 1/2
The point-like constituents of the nucleon have spin 1/2
Hadrons are complicated objects: they consist of quarks, anti-quarks and gluons (partons)

- **Valence quarks**: determine quantum numbers of hadron
- **Sea quarks**: virtual quark-antiquark pairs produced from gluons
See you on Wednesday 02/25!

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Or

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