Lecture 25:
6. Nuclear decays: \( \gamma \) (lecture 24) \( \alpha, \beta, \)
7. Nuclear fission and fusion
The Valley of Stability

The largest numbers of stable nuclei are near “magic numbers”
Radioactivity – Statistical Aspect

N - number of atoms of unstable isotope at the instant t
λ – decay constant

\[ dN(t) = -\lambda N dt \]

\[ N(t) = N_0 e^{-\lambda t} \]

where \( N_0 \) - number of atoms of unstable isotope at the initial time \( t_0 = 0 \)

The survival probability \( S(t) \) that the radioisotope does not decay during an arbitrary time interval \( t \)

\[ S(t) = N(t)/N_0 = e^{-\lambda t} \]

\( t_{1/2} \) - half-live of a radioactive sample (time needed for the decay of half of the atoms of the sample)

\[ t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \]

\( \tau \) – mean lifetime of a radioactive sample (time needed for the decay to \( 1/e \) of the initial value of the atoms of the sample)

\[ \tau = 1/\lambda \]
Radioactivity – Units

\[
-\frac{dN(t)}{dt} = \lambda N \equiv A(t)
\]

Activity

Standard unit of radioactivity:

- the curie [Ci]
  \[1 \text{ Ci} = 3.7 \times 10^{10} \text{ desintegrations / sec}\]
  \(\sim\) the activity of 1 gram of \(^{226}\text{Ra}\)

- the becquerel [Bq]
  \[1 \text{ Bq} = 2.7 \times 10^{-11} \text{ Ci} \sim 1 \text{ desintegration / sec}\]
Alpha Decays

- Most radioactive nuclides are $\alpha$- emitters ($A>150$)

- The decay constant: $\lambda_a \sim e^{-c/v} = e^{-c/\sqrt{Q}}$
  
  $v$ - Relative speed of a particle and the daughter nucleus
  
  $Q \sim v^2$ - decay energy

$$Q = \{M(\frac{A}{2}X) - M(\frac{A-4}{2}Y) - M(\frac{4}{2}He)\}c^2$$
Alpha Decays

- The energy and momentum conservation:

\[ p_D + p_\alpha = 0 \]

\[ M_PC^2 = (M_DC^2 + T_D) + (M_\alpha C^2 + T_\alpha) \]

In the non-relativistic limit:

\[ T_D = p_D^2 / 2M_D = p_\alpha^2 / 2M_\alpha = \left( M_\alpha / M_D \right) T_\alpha \]

\[ Q = [M_P - (M_D + M_\alpha)]C^2 = T_D + T_\alpha = \frac{M_D + M_\alpha}{M_D} T_\alpha \approx \frac{A}{A - 4} T_\alpha \implies Q > T_\alpha \]

- \( \alpha \) energy uniquely determined by masses and Q value

- \( \alpha \)-decay – an exothermic process ("outside heating")
  i.e. a process or reaction that releases energy from the system
**Alpha Decays**

- Most radioactive nuclides are $\alpha$-emitters ($A>150$)
  \[
  \frac{A}{Z}P \rightarrow \frac{A-4}{Z-2}D + \frac{4}{2}He + Q
  \]
  \[
  Q = [M_P - M_D - M_\alpha]c^2
  \]

- **Spontaneous $\alpha$ decay** $Q > 0$
  \[
  Q \sim 3-11 \text{ MeV} \quad \text{for known $\alpha$ emitters}
  \]

\[
Q = T_\alpha + T_D
\]
\[
Q \approx T_\alpha \frac{A}{A-4} (A \gg 4)
\]
\[
\Rightarrow T_D = Q - T_\alpha \approx Q \frac{4}{A}
\]

$A \sim 200 : T_\alpha \sim 0.98Q, \ T_D \sim 0.02Q$
Alpha Decays

Geiger-Nuttall rule
universal behavior

How to explain it?

\[ \log(t_{1/2}) = a + \frac{b}{\sqrt{Q_\alpha}} \]
Energetics of alpha decay

\[ E_{\text{coul}} = \frac{e^2 Z_\alpha Z_Y}{4\pi\varepsilon_0} \frac{1}{R} = \frac{e^2 Z_\alpha Z_Y}{4\pi\varepsilon_0} \frac{1}{r_0(A^{1/3}_\alpha + A^{1/3}_Y)} = \alpha \hbar c \frac{Z_\alpha Z_Y}{r_0(A^{1/3}_\alpha + A^{1/3}_Y)} \]

\[ ^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^4_2\text{He} \]

\[ \alpha \hbar c \frac{Z_\alpha Z_Y}{r_0(A^{1/3}_\alpha + A^{1/3}_Y)} = \frac{1}{137} \cdot 197.32 \frac{2.86}{1.2(4^{1/3} + 222^{1/3})} \approx 25 \text{ MeV} \]

\[ Q = (226.025408 - 222.017526 - 4.0026)u \approx 4.9 \text{ MeV} \]
Tunneling of an \( \alpha \) – particle through a nuclear Coulomb barrier.

\[
\lambda_\alpha \sim \left( \frac{v}{R} \right) P
\]

- \( R \)-radius of the daughter nucleus
- \( P \)-transmission coefficient

The prefactor \( v / R \) is the attempt frequency, the rate at which the particle tries to tunnel through the barrier, and \( P \) is the probability of tunneling for each try.

Gamow factor \( G \)

\[
\ell n(t_{1/2}) = \ell n(0.693R / v) + 4\pi Z_D e^2 / \hbar v - \frac{8}{\hbar} \left( Z_D e^2 \mu R \right)^{1/2}
\]

\[
R \sim A^{1/3} \sim Z_D^{1/3}
\]

\[
\log(t_{1/2}) = a + \frac{b}{\sqrt{Q_\alpha}}
\]
Spin and Parity in $\alpha$ decay

- Conservation of angular momentum:
  \[ \vec{J}_P = \vec{J}_D + \vec{J}_\alpha + \vec{L} \]
  where:
  \( J_P (J_D) \) – the spin of the parent (daughter)
  \( L \) - the orbital angular momentum in the center of mass

- Conservation of parity:
  \[ \pi(P) = \pi(D)\pi(\alpha)(-1)^l \]

  \[ \pi(P) = \pi(D)(-1)^l \]  
  since  \( J^\pi(\alpha) = 0^+ \)
Spin and Parity in $\alpha$ decay

**Example:** Emission of the spinless $\alpha$ particle from the spin 0+ ground state in even-even nuclei:

\[
J_P = 0, \ J_\alpha = 0 \Rightarrow \vec{J}_D = -\vec{L} \\
\pi(D) = (-1)^l
\]

- Relation between the spin/parity of the state populated by the decay in the daughter nucleus and the angular momentum channel of the decay.
- The only states which can be populated by $\alpha$ decay from ground states of even-even nuclei are these of **even spin and positive parity** or **odd spin and negative parity**.
Branching ratios of $\alpha$-decay to excited states of even-even nuclei

Example: $^{242}\text{Cm} \rightarrow ^{238}\text{Pu} + \alpha$

Final states of the daughter, $^{238}\text{Pu}$:

http://arxiv.org/abs/0902.0267

- Excited states of $^{238}\text{Pu}$ are not populated with the same probability.
- For most of the time the decay populates the ground state in the daughter.

Question: The two states 2- and 3+ are not populated (experimental result). Why?
Branching ratios of $\alpha$-decay to excited states of odd nuclei

$$J_P = J_D + J_\alpha + L$$

if $j_P \neq 0 \neq j_D \Rightarrow |j_P - j_D| \leq l \leq j_P + j_D \quad \pi(P) = \pi(D)\pi(\alpha)(-1)^l$

Es = Einsteinium (Z=99)
Bk = Berkelium (Z=97)

$l = \text{even}$

$4 \leq l \leq 11$

$3 \leq l \leq 10$

$2 \leq l \leq 9$

$1 \leq l \leq 8$

$0 \leq l \leq 7$

Figure 7.6 Intensities (%) for the several components of angular moment in the $\alpha$-decay of $^{253}$Es to $^{249}$Be. For $l \geq 8$ the intensities are very small and not available to measurement [So70].
Beta Decays

- The most common form of radioactive disintegration (all nuclides not lying in the “valley of stability” are unstable against this transition)

\[ {^A_Z P \rightarrow {^{A+1}_Z D + e^\mp + \bar{\nu}_e + Q} \]

- **β⁻ decay**
  - At partonic level
  - \[ n \rightarrow p + e^- + \bar{\nu}_e \]
  - \[ d \rightarrow u + W^- \rightarrow u + e^- + \bar{\nu}_e \]

- **β⁺ decay**
  - \[ p_{(bound)} \rightarrow n + e^+ + \nu_e \]
  - \[ u \rightarrow d + W^+ \rightarrow d + e^+ + \nu_e \]

- **electron capture**
  - \[ e^- + p_{(bound)} \rightarrow n + \bar{\nu}_e \]

\[ W^\pm, Z^0 \text{ quanta of electroweak force} \]

\[ M_W \sim 80\text{GeV} \]
\[ M_Z \sim 90\text{GeV} \]

- Free proton does not decay but possible in nuclei if Q value is right
\[ M_{\text{atomic}} c^2 = Mc^2 + Zm_e c^2 - \sum_{i=1}^{Z} B_{ei} \]

- **β⁻ decay**
  \[ {}_Z^A P \to {}_{Z+1}^A D + e^- + \bar{\nu}_e + Q_{\beta^-} \]
  \[ Q_{\beta^-} = T_e + T_\nu = \left[ M_P - M_D - m_e \right] c^2 \]
  differences in the parent and the daughter electron binding energies can be neglected
  \[ Q_{\beta^-} = \left[ M_P^{\text{atomic}} - M_D^{\text{atomic}} \right] c^2 \quad M_P^{\text{atomic}} > M_D^{\text{atomic}} \Rightarrow Q_{\beta^-} > 0 \]

- **β⁺ decay**
  \[ {}_Z^A P \to {}_{Z-1}^A D + e^+ + \nu_e + Q_{\beta^+} \]
  \[ Q_{\beta^+} = T_e + T_\nu = \left[ M_P - M_D - m_e \right] c^2 \]
  \[ Q_{\beta^+} = \left[ M_P^{\text{atomic}} - \left( M_D^{\text{atomic}} + 2m_e \right) \right] c^2 \quad M_P^{\text{atomic}} > M_D^{\text{atomic}} + 2m_e \Rightarrow Q_{\beta^-} > 0 \]

- **Electron capture**
  \[ {}_Z^A P + e^- \to {}_{Z+1}^A D + \nu_e + Q_{EC} \]
  \[ Q_{EC} = \left[ M_P^{\text{atomic}} - \left( M_D^{\text{atomic}} - B_{en} \right) \right] c^2 \]
  \[ n = K, L_I, L_{II}, \ldots \]
The atomic masses for Parent-Daughter systems illustrating the energy relations in $\beta^-$ and $\beta^+$ (EC) decay processes. Various Q-values are indicated with $2m_e c^2$ (~1.022 MeV) as a threshold for $\beta^+$ decay.

Examples:

<table>
<thead>
<tr>
<th>Decay</th>
<th>Type</th>
<th>$Q$ (MeV)</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{23}$Ne $\rightarrow$ $^{23}$Na + $e^-$ + $\bar{\nu}_e$</td>
<td>$\beta^-$</td>
<td>4.38</td>
<td>38 s</td>
</tr>
<tr>
<td>$^{99}$Tc $\rightarrow$ $^{99}$Ru + $e^-$ + $\bar{\nu}_e$</td>
<td>$\beta^-$</td>
<td>0.29</td>
<td>$2.1 \times 10^5$ y</td>
</tr>
<tr>
<td>$^{25}$Al $\rightarrow$ $^{25}$Mg + $e^+$ + $\nu_e$</td>
<td>$\beta^+$</td>
<td>3.26</td>
<td>7.2 s</td>
</tr>
<tr>
<td>$^{124}$I $\rightarrow$ $^{124}$Te + $e^+$ + $\nu_e$</td>
<td>$\beta^+$</td>
<td>2.14</td>
<td>4.2 s</td>
</tr>
<tr>
<td>$^{15}$O + $e^-$ $\rightarrow$ $^{15}$N + $\nu_e$</td>
<td>EC</td>
<td>2.75</td>
<td>1.22 s</td>
</tr>
<tr>
<td>$^{41}$Ca + $e^-$ $\rightarrow$ $^{41}$K +$\nu_e$</td>
<td>EC</td>
<td>0.43</td>
<td>$1.0 \times 10^5$ y</td>
</tr>
</tbody>
</table>
Nuclear beta decay: Fermi theory  
(pre- Standard Model)

Historically the prototype weak interaction was nuclear beta decay

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

At the low momentum transfers involved \( q \ll M_W \) the interaction is point-like and described by the four-fermion coupling \( G \).

The transition probability (decay rate per unit time) from one energy eigenstate of a quantum system \( |i\rangle \) into a continuum of energy eigenstates \( |f\rangle \). It is related to mean lifetime.

\[ \lambda = \frac{2\pi}{\hbar} G^2 |M_{fi}|^2 \frac{dn}{dE} \]

\( E \) – energy in the final state
\( \frac{dn}{dE} \) – the density of final states per unit energy interval
\( |M_{fi}|^2 \) - the square matrix element, involving integration over angles and spin directions of the particles concerned

\[ |M_{fi}|^2 = \left| \langle \psi_f | H_{\text{int}} | \psi_i \rangle \right|^2 \]

(the original Fermi theory does not take into account the spins of the particles) (vector sum)

If the total angular momentum (summed over the lepton spins) is

- \( J(\text{leptons})=0 \) (antiparallel) then \( |M|^2 \sim 1 \) (Fermi transitions)
- \( J(\text{leptons})=1 \) (parallel) then \( |M|^2 \sim 3 \) (Gamow-Teller transitions)
The V-A interaction
Fermi and Gammow-Teller Transitions

Weak decays do not conserve parity: all the leptons emitted in beta-decays are left-handed (negative helicity) and all anti-leptons - right-handed (positive helicity):

\[ h(\nu_e) \approx -1, \quad h(\bar{\nu}_e) \approx +1, \quad h(e^\pm) \approx \pm \frac{V}{c}, \]

\[ h(\nu_e) \approx -1, \quad h(\bar{\nu}_e) \approx +1, \quad h(e^\pm) \approx \pm \frac{V}{c}, \]

\[ \frac{g^2}{8M_W^2} \equiv \frac{G}{\sqrt{2}} \]

- In Fermi theory of weak interactions, before parity violation was discovered, the operator \( O \) was assumed to be a vector operator (the same as in e-m).
- The discovery of parity violation implied a combination of two types of interaction with opposite parities: \( V \) (vector, odd parity) and \( A \) (axial vector, even parity).
- A general combination of \( V \) and \( A \) amplitudes would correspond to a operator of the form:

\[ GJ^\text{weak}_{\text{baryon}}J^\text{weak}_{\text{lepton}} \rightarrow \sum_{j=1}^{A} \left[ G_V i_\pm(j) + G_A \bar{\sigma}(j) \cdot \bar{\tau}(j) \right] \]

\[ \text{Gamow-Teller decay} \]

\[ G_V \sum_{j=1}^{A} \left[ i_\pm(j) + g_A \bar{\sigma}(j) \cdot \bar{\tau}(j) \right] \]

\[ \text{Fermi decay} \]

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<th>Transition</th>
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<td>( \Lambda \beta ) decay</td>
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<tr>
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Fermi and Gammow-Teller Transitions

\[ I_P = I_D + l + S \]
\[ \pi^P = \pi^D (-1)^l \]

- **Fermi decay:**
  \[ I_P = I_D + l \]
  the charged lepton and the neutrino have anti-parallel spins, thus do not contribute to the angular momentum balance

- **Gamow-Teller decay:**
  \[ I_P = I_D + l + 1 \]
  the charged lepton and the neutrino have parallel spins, thus contribute one unit to the angular momentum balance

The Gamow-Teller transitions are not accounted for by Fermi theory, which do not take into account the spins of the particles.

- \( l = 0 \)  Allowed
- \( l = 1 \)  First forbidden
- \( l = 2 \)  Second forbidden
Allowed $\beta$ Decay (l=0)

- **Fermi decay:**

\[
l = 0 \Rightarrow I_P = I_D \quad \Rightarrow \quad \Delta I = 0
\]

\[
\pi^P = \pi^D \quad \Rightarrow \Delta \pi = 0
\]

\[
\left| M'_{ij} \right|_{Fermi}^2 = 1
\]

\[
I_P = 0, I_D = 0 \quad \text{superallowed}
\]

- **Gamow-Teller decay:**

\[
l = 0 \Rightarrow I_P = I_D + 1 \quad \Rightarrow \quad |\Delta I| = 0, 1
\]

\[
\pi^P = \pi^D \quad \Rightarrow \Delta \pi = 0
\]

\[
\left| M'_{ij} \right|_{Gamow-Teller}^2 = 3
\]

\[
I_P = 0, I_D = 0 \quad \text{forbidden}
\]

Allowed Fermi and Gamow-Teller transitions have a maximum angular momentum change of one unit and no change in parity.
### Classification in $\beta$-decay

\[
\tilde{J}_p = \tilde{J}_D + \tilde{L}_\beta + \tilde{S}_\beta \\
\pi_p = \pi_D(-1)^{L_\beta}
\]

<table>
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<tr>
<th>Allowed transitions</th>
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<tr>
<td>1st forbidden transitions</td>
<td>$\tilde{L}_\beta = 1$</td>
</tr>
<tr>
<td>2nd forbidden transitions</td>
<td>$\tilde{L}_\beta = 2$</td>
</tr>
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<td>$\tilde{S}_\beta = 0$</td>
</tr>
<tr>
<td>Gamow–Teller transitions</td>
<td>$\tilde{S}_\beta = 1$</td>
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#### I. Allowed transitions ($\tilde{L}_\beta = 0, \pi_p = \pi_D$)

- Fermi-type ($\tilde{S}_\beta = 0$)
  - $\tilde{J}_p = \tilde{J}_D$
  - $|\Delta J| = 0$
  - $0^+ \rightarrow 0^+$: superallowed

- Gamow–Teller type ($\tilde{S}_\beta = 1$)
  - $\tilde{J}_p = \tilde{J}_D + 1$
  - $|\Delta J| = 0, 1$: no $0^+ \rightarrow 0^+$
  - $0^+ \rightarrow 1^+$: unique Gamow–Teller

#### II. 1st forbidden transitions ($\tilde{L}_\beta = 1, \pi_p = -\pi_D$)

- Fermi-type ($\tilde{S}_\beta = 0$)
  - $\tilde{J}_p = \tilde{J}_D + 1$
  - $|\Delta J| = 0, 1$
  - no $0^- \rightarrow 0^+$

- Gamow–Teller type ($\tilde{S}_\beta = 1$)
  - $\tilde{J}_p = \tilde{J}_D + 1 + 1$
  - $|\Delta J| = 0, 1, 2$
  - 3 types
    - (i) $|\Delta J| = 0$
    - (ii) $|\Delta J| = 0, 1$;
      - no $0^+ \rightarrow 0^+$
    - (iii) $|\Delta J| = 0, 1, 2$;
      - no $1^+ \rightarrow 0^-$
      - no $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$
Exercise:

Determine the type of transitions:
- Fermi (F) and/or Gamow-Teller (GT)
- allowed, first forbidden, second forbidden

for the following processes:

the lowest order transitions:

\[ n\left(\frac{1}{2}^+\right) \rightarrow p\left(\frac{1}{2}^+\right) \quad \pi_i=\pi_j \text{ (no parity change) } \Delta J=0, \text{ allowed (F and GT)} \]

\[ ^{38}_{17}Cl\left(2^-\right) \rightarrow ^{38}_{18}Ar\left(0^+\right) \quad \pi_i=-\pi_j \text{ (parity change) } \Delta J=2, \text{ 1\textsuperscript{st} forbidden (GT)} \]

\[ ^{38}_{17}Cl\left(2^-\right) \rightarrow ^{38}_{18}Ar\left(2^+\right) \quad \pi_i=-\pi_j \text{ (parity change) } \Delta J=0, \text{ 1\textsuperscript{st} forbidden (F and GT)} \]

\[ ^{38}_{17}Cl\left(2^-\right) \rightarrow ^{38}_{18}Ar\left(3^-\right) \quad \pi_i=\pi_j \text{ (no parity change) } \Delta J=1, \text{ allowed (GT)} \]

\[ ^7_{4}Be\left(\frac{3}{2}^-\right) \rightarrow ^7_{3}Li\left(\frac{3}{2}^-\right) \]

\[ ^7_{4}Be\left(\frac{3}{2}^-\right) \rightarrow ^7_{3}Li\left(\frac{1}{2}^-\right) \]

\[ ^{64}_{29}Cu\left(1^+\right) \rightarrow ^{64}_{30}Zn\left(0^+\right) \]

\[ ^{64}_{29}Cu\left(1^+\right) \rightarrow ^{64}_{28}Ni\left(0^+\right) \]
Figure 5-8: Systematics of observed $\log_{10} fT$ values. The grey area in the upper panel shows 718 cases of $0^+ \rightarrow 1^+$ allowed transitions, and the remaining 1741 cases of other allowed decays are shown by the white histogram. The peak of the distribution for the 24 cases of $0^+ \rightarrow 0^+$ superallowed decay is indicated by the arrow. The 216 first-forbidden unique transitions ($|J_i - J_f| = 2$) are shown by the shaded part in the lower panel and the 1086 cases of other first-forbidden transitions by the white histogram. Only 37 second-forbidden and 3 third-forbidden cases are known. The four reported cases of the highest order, fourth, have $\log_{10} fT$ values above 20. (Based on data in Ref. [79], selected by Singh et al. [128].)
Nuclear Fission

- Discovered in 1939
- a decay into lighter nuclei. Exothermic reaction. It can release the total energy of ~1 MeV per nucleon (e-m radiation + kinetic energy of the fragments). Thus the total binding energy of the resulting elements must be larger than that of the starting element.

Types of fission:

- spontaneous (tunneling effect)
  
  [External Link]
  
  http://en.wikipedia.org/wiki/Spontaneous_fission

- induced (nuclear reactions)
- The largest binding energy per nucleon for nuclei $A \sim 56$ (iron group)
- $A > 110-120$: binding energy decreases and a nucleus can split into 2 nuclei
- Uranium isotopes are the lightest nuclei for which the probability of spontaneous fission is of the same order as for α decay ($A > \sim 230$).
Spontaneous Nuclear Fission

Spontaneous fission is due to tunneling through the fission barrier.

Figure 13.1 Inside its nuclear potential well, $^{238}$U may perhaps exist instantaneously as two fragments of $^{190}$Pd, but the Coulomb barrier prevents them from separating.
Nuclear Fission Energy

Fission energy is the release energy from the fission of the nucleus of mass \( M(A,Z) \) to fragments with masses \( M_1(A_1,Z_1) \) and \( M_2(A_2,Z_2) \):

\[
Q_f = M(A,Z)c^2 - \left[ M_1(A_1,Z_1)c^2 + M_2(A_2,Z_2)c^2 \right] \\
= W_1(A_1,Z_1) + W_2(A_2,Z_2) - W(A,Z) \\
W = E_B
\]
Nuclear Fission Energy

Fission energy is the release energy from the fission of the nucleus of mass \( M(A,Z) \) to fragments with masses \( M_1(A_1,Z_1) \) and \( M_2(A_2,Z_2) \):

\[
Q_f = M(A,Z)c^2 - \left[ M_1(A_1,Z_1)c^2 + M_2(A_2,Z_2)c^2 \right] \\
= W_1(A_1,Z_1) + W_2(A_2,Z_2) - W(A,Z) \\
W = E_B
\]

- **Symmetric fission**
  
  \[ M_1(A_1,Z_1) = M_2(A_2,Z_2) = M(A/2,Z/2) \]
  
  \[ Q_f = 2W(A/2,Z/2) - W(A,Z) \approx \left[ E_s(A,Z) + E_c(A,Z) \right] - 2\left[ E_s(A/2,Z/2) + E_c(A/2,Z/2) \right] \]
  
  Fission is energetically favorable if \( Q_f > 0 \): \( \frac{Z^2}{A} \geq 17 \) (for nuclei with \( A > 90 \))

- **Asymmetric fission**
  
  Fission products: \( A(\text{light})=95\pm15 \) and \( A(\text{heavy})=135\pm15 \)
  
  (closed shells for the products)

\[
\frac{A(\text{light})}{A(\text{heavy})} \approx \frac{Z(\text{light})}{Z(\text{heavy})} \approx \frac{2}{3}
\]

\[
n + ^{235}_{92}\text{U} \rightarrow ^{236}_{92}\text{U} \rightarrow ^{95}_{38}\text{Sr} + ^{139}_{54}\text{Xe} + 2n
\]
Nuclear Fission - Mechanism

- the fission of a heavy nucleus requires a total input energy of ~7-8 MeV to initially overcome the strong force which holds the nucleus into a spherical or nearly spherical shape

- deform it into a “peanut” shape

- separation to a critical distance, beyond which the strong force is too weak

- separation process proceeds by the energy of the e-m repulsion between the fragments
Fig. 3.8. Potential energy during different stages of a fission reaction. A nucleus with charge Z decays spontaneously into two daughter nuclei. The solid line corresponds to the shape of the potential in the parent nucleus. The height of the barrier for fission determines the probability of spontaneous fission. The fission barrier disappears for nuclei with $Z^2/A \gtrsim 48$ and the shape of the potential then corresponds to the dashed line.
Let's estimate the $Z$ value above which nuclei are fission unstable by comparing the surface and Coulomb energies during the fission deformation.

\[
\begin{align*}
    a &= R(1 + \varepsilon) \\
    b &= R(1 - \varepsilon/2)
\end{align*}
\]

**Fig. 3.9.** Deformation of a heavy nucleus. For a constant volume $V (V = 4\pi R^3/3 = 4\pi ab^2/3)$, the surface energy of the nucleus increases and its Coulomb energy decreases.

\[
\begin{align*}
    E_{\text{surface}} &= a_s A^{2/3} \left( 1 + \frac{2}{5} \varepsilon^2 + \ldots \right), \\
    E_{\text{Coulomb}} &= a_c Z^2 A^{-1/3} \left( 1 - \frac{1}{5} \varepsilon^2 + \ldots \right)
\end{align*}
\]

A deformation changes the total energy by $\Delta E$:

\[
\Delta E = \frac{\varepsilon^2}{5} \left( 2a_s A^{2/3} - a_c Z^2 A^{-1/3} \right)
\]

$\Delta E < 0 : \ 2a_s A^{2/3} - a_c Z^2 A^{-1/3} < 0$

\[
\frac{Z^2}{A} \geq \left( \frac{Z^2}{A} \right)_{\text{critical}} = \frac{2a_s}{a_c} \sim 48 \quad (Z > 114 \ \text{and} \ A > 270)
\]

\[
\begin{align*}
    x &= \frac{Z^2/A}{\left( Z^2/A \right)_{\text{critical}}} \\
    x > 1 & \text{ immediate fission} \\
    x < 1 & \text{ stable}
\end{align*}
\]
Nuclear Fission Barrier

- $Q_f$ - fission energy
- $Z^2/A$ - fission parameter
- $U$ - fission barrier
  
  \[ U(r) = V_{\text{max}} - V_{r=0} \]

Fission energetically favored for $Q_f > 0$:

- e.g. $^{235}\text{U}$
  - $Z > 114$ and $A > 270$

\[ Q_f > 0 \]
Induced Fission

Barium-142 and krypton-91 are two possible fission fragments produced when a neutron is absorbed by uranium-235, causing a fission reaction.

\[ n + U^{235} \rightarrow Kr^{91} + Ba^{142} + 3n \]
Nuclear Fission Chain Reaction

- It occurs when one nuclear reaction causes on the average one or more nuclear reactions, leading to a self-propagating number of these reactions.

- The production of neutrons in each fission event makes it possible to use fission chain reactions for the production of energy.

1st Generation: on average 2 neutrons

\[ \ldots \]

\[ k^{\text{th}} \text{ Generation: } 2^k \text{ neutrons} \]

- Nuclear fuels: the chemical element isotopes that can sustain a fission chain reaction (most common fuels: \(^{235}\text{U},^{239}\text{Pu}\))
Isotopic Enrichment

Natural Uranium: 0.7% U235 (0.7 billion years)

99.3% mostly U238 (4.5 billion years)

Only U235 is fissile, releasing neutrons spontaneously. These neutrons are too fast to cause a chain reaction at this concentration.

If they are slowed down, chain reaction is adequate for a nuclear power plant.

Slow (thermal) neutrons: reduction of the neutron kinetic energy takes place by transfer of energy to a “moderator”.

If the U235 is enriched to maybe 7%, then the fast neutrons can cause a chain reaction. If there is a critical mass, a nuclear bomb is possible.
Critical Mass

- For a chain reaction of nuclear fission (e.g. uranium-235) is to sustain itself, then at least one neutron from each fission must strike another U-235 nucleus and cause a fission.

- If this condition is just met, then the reaction is said to be "critical" and will continue. The mass of fissile material required to achieve this critical condition is said to be a critical mass.

- The critical mass depends upon the concentration of U-235 nuclei in the fuel material as well as its geometry.

As applied for the generation of electric energy in nuclear reactors, it also depends upon the moderation used to slow down the neutrons. In those reactors, the critical condition also depends upon neutrons from the fission fragments, called delayed neutrons. For weapons applications, the concentration U-235 must be much higher to create a condition called "prompt criticality". This means that it is critical with only the neutrons directly produced in the fission process. For U-235 enriched to "bomb-grade" uranium, the critical mass may be as small as about 15 kg in a bomb configuration.
There are only three known nuclides (arrangements of protons and neutrons) that undergo fission when introduced to a slow (thermal) neutron:

- $^{233}$U: hardly used (hard to get/make)
- $^{235}$U: primary fuel for reactors
- $^{239}$Pu: popular in bombs; also breeder reactors
  - When neutron hits a $^{238}$U it turns into $^{239}$Pu

Other nuclei may split if smacked hard enough by a neutron (or other energetic particle)

**$^{235}$U bomb is extremely easy to make**

- Just make ball of 235U above critical mass it will explode. Easiest form is “gun” bomb

U.S. made 3 bombs:

1. Tested in New Mexico, July 1945: 6 kg of $^{239}$Pu
2. Hiroshima: 60 kg of 235U (gun type) (equiv to 12,000 tons TNT)
3. Nagasaki: 6 kg of $^{239}$Pu (equiv to 22,000 tons of TNT)

TNT (Trinitrotoluene): the explosive yield of TNT is considered to be the standard measure of strength of bombs and other explosives.
Graphite was used to moderate to slow down the neutrons. Control rods set the number of neutrons. This is a “mechanically” controlled system.

Example: Chernobyl type Reactor

- Simple concept: need exactly one excess neutron per fission event to find another $^{235}\text{U}$
- Inserting a neutron absorber into the core removes neutrons from the pool
- Pulling out rod makes more neutrons available
- Emergency procedure is to *drop* all control rods at once

Can also have fusion: even more powerful

- Helium nucleus is lighter than the four protons!
- Mass difference is $4.029 - 4.0015 = 0.0276$ a.m.u.
- 0.7% of mass is converted into energy via $E=mc^2$
- Reaction ~20 million times more energetic than chemical reactions, in general
  - Fusion works out to 150 million Calories per gram
  - compare to 16 million Cal/g uranium fission
  - Compare to 10 Cal/g gasoline

\[ \begin{align*}
\text{4 protons:} & \quad \text{mass} = 4.029 \\
\rightarrow & \quad 4^\text{He nucleus:} \\
& \quad \text{mass} = 4.0015 \\
& \quad + 2 \text{ neutrinos, photons (light)} \\
& \quad + 26 \text{ MeV of energy}
\end{align*} \]

THIS FUSION IS THE SOURCE OF THE SUN’S ENERGY, (and of energy in thermonuclear hydrogen bombs)
See you at the Final!

Questions, comments:
e-mail: Joanna.Kiryluk (at) stonybrook.edu

Or

C109 (Physics Building)

Final Exam:  Sunday May 17 10am-12.30pm
we’ll meet near library C floor.