Lecture 17:
4. Nuclear Force

Textbook (S. Wong, chapter 3)

2nd Midterm April 15 (Wednesday)
The two-body nuclear potential

Phenomenological method: use appropriate form for the potential with a sufficient amount of parameters.

Most general form: potential is a function of the relevant degrees of freedom of two nucleons:

\[ V(1,2) = \left( r_j, p_j, \sigma_j, i_j ; j = 1,2 \right) \]

The force between nucleons must be invariant under a translation, rotation, Galilean transformations, and particle exchange: \[ V(1,2) = V(2,1) \]

It can only depend on relative position and momentum of 2 nucleons:

\[ r = r_1 - r_2 \]
\[ p = \frac{1}{2} (p_1 - p_2) \]

2-nucleons potential: \[ V(1,2) = \left( r, p, \sigma_j, i_j ; j = 1,2 \right) \]
Nuclear Potential

2-nucleons potential:

\[ V(1,2) = \left( r, p, \sigma_j, i_j, j = 1, 2 \right) \]

Lecture 16 exercise:
Calculate the expectation value of \( \sigma_1 \cdot \sigma_2 \) (index 1,2 means nucleon 1, 2) in the space of two nucleons with total spin S:

\[
\left\langle \sigma_1 \cdot \sigma_2 \right\rangle =
\begin{cases} 
-3 & (S=0) \\
=+1 & (S=1)
\end{cases}
\]

\( \Rightarrow \sigma_1 \cdot \sigma_2 \) operator is able to distinguish between S=0 and S=1 states

Similarly the isospin …

Are \( \sigma_1 \cdot \sigma_2 \) and \( \hat{i}_1 \cdot \hat{i}_2 \) the only “good” combinations?
Rotational invariance

- determines the structure of the spin degrees of freedom

\[ f(\sigma_1, \sigma_2) \] classification by tensor properties:

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“Scalar” = rank 0 tensor, “Vector” = rank 1 tensor, “Tensor” = rank 2

The indices [1] and [2] refer the coupling scheme of two tensor operators \( T_1^{[L_1]} \) and \( T_2^{[L_2]} \) into a new operator:

\[
T_{[m]}^{[L]} = \left[ T_1^{[L_1]} \times T_2^{[L_2]} \right]_{[m]}^{[L]} = \sum_{m_1m_2} \langle L_1m_1, L_2m_2 | Lm \rangle T_{[m_1]}^{[L_1]} T_{[m_2]}^{[L_2]} 
\]

Wang (textbook)
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\]

Wang (textbook)
Pauli matrices:

\[ \sigma = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z} \]

\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

can be used to define operators:

\[ \sigma_\pm = \frac{1}{2} (\sigma_x \pm i \sigma_y) \]

\[ \sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \]

rising/lowering operators

\[ \sigma_+ \chi_+ = 0 \]
\[ \sigma_+ \chi_- = \chi_+ \]
\[ \sigma_- \chi_+ = \chi_- \]
\[ \sigma_- \chi_- = 0 \]
Pauli matrices: Operators which acts on intrinsic spin part of the wave function

\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]
\[ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]
\[ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

These form 3 components of an operator acting on the intrinsic spin part of the wave function of a nucleon and carrying one unit of angular momentum (similar to other vector operators such as \( \mathbf{L} \))

Notation:

\[ \sigma_{+1} = -\frac{1}{\sqrt{2}}(\sigma_x + i\sigma_y) \]
\[ \sigma_{-1} = \frac{1}{\sqrt{2}}(\sigma_x - i\sigma_y) \]
\[ \sigma_0 = \sigma_z \]

Spherical tensor operator, Appendix A

\[ \sigma_{+1} = -\sqrt{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \]
\[ \sigma_{-1} = \sqrt{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \]
\[ \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]
Pauli matrices: Operators which acts on intrinsic spin part of the wave function

\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

These form 3 components of an operator acting on the intrinsic spin part of the wave function of a nucleon and carrying one unit of angular momentum (similar to other vector operators such as \( \mathbf{L} \)).

\[ \sigma_+ = -\frac{1}{\sqrt{2}} \left( \sigma_x + i\sigma_y \right), \quad \sigma_- = \frac{1}{\sqrt{2}} \left( \sigma_x - i\sigma_y \right) \]

\[ \sigma_0 = \sigma_z \]

Spherical tensor operator, Appendix A

\[ \left[ V_i, J_j \right] = i\epsilon_{ijk} V_k \]

\[ T_0^1 = V_z, T_{\pm1}^1 = \pm \frac{1}{\sqrt{2}} \left( V_x \pm iV_y \right) \]

\( V \) – spherical tensor rank 1

We can check that this is the same as scalar products defined in terms of Cartesian components of the vectors by noting that

\[ J_{\pm1} = \pm \frac{1}{\sqrt{2}} (J_x \pm iJ_y) \]

\[ V_{\pm1} = \pm \frac{1}{\sqrt{2}} (V_x \pm iV_y) \]

This is slightly different from the definition of angular momentum raising and lowering operators \( \mathbf{L}_\pm = L_x \pm iL_y \) as the usual convention does not attempt to make them spherical tensor operators.
Exercise: check that 1) and 2) gives the same answer

1)
\[ \sigma(1) \cdot \sigma(2) = \sigma_x(1) \cdot \sigma_x(2) + \sigma_y(1) \cdot \sigma_y(2) + \sigma_z(1) \cdot \sigma_z(2) \]

2) \[ J \cdot V = \sum_q (-1)^q J_{1q} V_{1,-q} \quad \text{(any J, V spherical tensors rank 1)} \]
\[ \sigma(1) \cdot \sigma(2) = \sigma_0(1) \cdot \sigma_0(2) - \sigma_{+1}(1) \cdot \sigma_{-1}(2) - \sigma_{-1}(1) \cdot \sigma_{+1}(2) \]
Spherical tensors

- tensors whose components have a specific behavior under spatial transformations.

- Example: 
\[ \vec{r} = (x, y, z) \] a space vector. The spherical components \( r_\mu, \mu = 0, \pm 1 \) of \( \vec{r} \) are:

\[ r_{+1} = -\frac{1}{\sqrt{2}}(x + iy), \quad r_{-1} = \frac{1}{\sqrt{2}}(x - iy) \]

\[ r_0 = z \]

A spherical tensor of rank one \( T^1_\mu \) is transformed under spatial rotations as vector \( r_\mu \)

- The Pauli matrices \( \vec{\sigma} \) correspond to a spherical tensor operator of rank one with the spherical components:

\[ \sigma_{+1} = -\frac{1}{\sqrt{2}}(\sigma_x + i\sigma_y), \quad \sigma_{-1} = \frac{1}{\sqrt{2}}(\sigma_x - i\sigma_y) \]

\[ \sigma_0 = \sigma_z \]

- Spherical tensors of higher rank are transformed like a product of coupled tensors of rank one. E.g. the spherical tensor of rank two is defined:

\[ T^{[2]}_{\mu} = \sum_{m_1m_2} \langle 1m_1, 1m_2 | 2\mu \rangle T^{[1]}_{[m_1]} T^{[1]}_{[m_2]} \]
Table 3-2: Values of Clebsch-Gordan coefficients \( \langle 1p1q|2m \rangle \).

| \( m \) | \( p \) | \( q \) | \( \langle 1p1q|2m \rangle \) | \( m \) | \( p \) | \( q \) | \( \langle 1p1q|2m \rangle \) |
|----|----|----|-----|----|----|----|-----|
| 0  | 1  | -1 | \( \sqrt{\frac{1}{6}} \) | 1  | 1  | 0  | \( \sqrt{\frac{1}{2}} \) |
| 0  | -1 | 1  | \( \sqrt{\frac{1}{6}} \) | 1  | 0  | 1  | \( \sqrt{\frac{1}{2}} \) |
| 0  | 0  | 0  | \( \sqrt{\frac{2}{3}} \) | -2 | -1 | -1 | 1   |
| -1 | -1 | 0  | \( \sqrt{\frac{1}{2}} \) | 2  | 1  | 1  | 1   |
| -1 | 0  | -1 | \( \sqrt{\frac{1}{2}} \) |    |    |    |     |

\[
T^{[2]}_{[\mu]} = \sum_{m_1m_2} \langle 1m_1, 1m_2 | 2\mu \rangle T^{[1]}_{[m_1]} T^{[1]}_{[m_2]}
\]

\[
(\sigma(1) \times \sigma(2))_{2m} = \sum_{pq} \langle 1p, 1q | 2m \rangle \sigma_p(1) \cdot \sigma_q(2)
\]

\[
(\sigma(1) \times \sigma(2))_{20} = \frac{1}{\sqrt{6}} \{ \sigma_1(1)\sigma_{-1}(2) + \sigma_{-1}(1)\sigma_1(2) + 2\sigma_0(1)\sigma_0(2) \}
\]

\[
(\sigma(1) \times \sigma(2))_{2\pm 1} = \frac{1}{\sqrt{2}} \{ \sigma_{\pm 1}(1)\sigma_0(2) + \sigma_0(1)\sigma_{\pm 1}(2) \}
\]

\[
(\sigma(1) \times \sigma(2))_{2\pm 2} = \sigma_{\pm 1}(1)\sigma_{\pm 1}(2)
\]
The two-body nuclear potential

One must be sure that the result (potential) is scalar and that symmetries under particle exchange, parity and time reversal are observed.

Possible combinations for $r$ and $p$:

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$$T^{[L]}_{[m]} = [T_1^{[L_1]} \times T_2^{[L_2]}]^{[L]}_{[m]} = \sum_{m_1m_2} \langle L_1m_1, L_2m_2 | Lm \rangle T^{[L_1]}_{[m_1]} T^{[L_2]}_{[m_2]}$$
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$$T_{[m]}^{[L]} = [T_{1}^{[L_1]} \times T_{2}^{[L_2]}]_{[m]}^{[L]} = \sum_{m_1 m_2} \langle L_1 m_1, L_2 m_2 | L m \rangle T_{[m_1]}^{[L_1]} T_{[m_2]}^{[L_2]}$$
Which vector-vector and tensor-tensor combinations (product must be a scalar) are possible?
The two-body nuclear potential

From symmetry and invariance properties, only the following vector-vector and tensor-tensor combinations are possible:

- **Vector-Vector**: spin-orbit operator

  \[
  L \cdot S = \frac{1}{2} (r \times p) \cdot (\sigma_1 + \sigma_2)
  \]

- **Tensor-Tensor**:

  \[
  \left[ \begin{array}{c} r^{[1]} \times r^{[1]} \end{array} \right]^{[2]} \cdot \left[ \begin{array}{c} \sigma^{[1]}_1 \times \sigma^{[1]}_2 \end{array} \right]^{[2]} = (\sigma_1 \cdot r)(\sigma_2 \cdot r) - \frac{1}{3}(\sigma_1 \cdot \sigma_2) r^2 \quad \rightarrow S_{12}(r)
  \]

  \[
  \left[ \begin{array}{c} p^{[1]} \times p^{[1]} \end{array} \right]^{[2]} \cdot \left[ \begin{array}{c} \sigma^{[1]}_1 \times \sigma^{[1]}_2 \end{array} \right]^{[2]} = (\sigma_1 \cdot p)(\sigma_2 \cdot p) - \frac{1}{3}(\sigma_1 \cdot \sigma_2) p^2 \quad \rightarrow S_{12}(p)
  \]

  \[
  \left[ \begin{array}{c} \sigma^{[1]}_1 \times \sigma^{[1]}_2 \end{array} \right]^{[2]} \cdot \left[ \begin{array}{c} r^{[1]} \times p^{[1]} \end{array} \right]^{[2]} \cdot p \quad \rightarrow (L \cdot S)^2
  \]

  \[
  (a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c).
  \]
The two-body nuclear potential

The most general form of the nuclear potential must be a function of 
\(\sigma_1, \sigma_2, i_1, i_2, r, p\) (spins, isospins, relative position, relative momentum):

\[
V(r; \sigma_1, \sigma_2, i_1, i_2) = V_C(r) + V_S(r)(\sigma_1 \cdot \sigma_2) \\
+ V_{LS}(r)(L \cdot S) \\
+ V_T(r)S_{12}(r) + V_{T'}(r)S_{12}(p) + V_Q(r)(L \cdot S)^2
\]

Central force 
Spin-orbit 
Tensor force 
non-central

\[
S_{12}(r) = \frac{3}{r^2}(\sigma_1 \cdot r)(\sigma_2 \cdot r) - (\sigma_1 \cdot \sigma_2)
\]

\[
S_{12}(p) = \frac{3}{p^2}(\sigma_1 \cdot p)(\sigma_2 \cdot p) - (\sigma_1 \cdot \sigma_2)
\]

\[
V_{\alpha} = V_{\alpha}(r^2, p^2, L^2) \text{ where } \alpha \in \{C, S, T, T', LS, Q\}
\]

Nucleon-nucleon interaction is independent of charge 
Isospin invariance: \(V_{\alpha}\) must be scalars in the isospin space, thus:

\[
V_{\alpha} = V_{\alpha0} + V_{\alpha1}(i_1 \cdot i_2)
\]
Potential between 2 nucleons

The central part of nucleon-nucleon potential

\[ V_C(r) = V_W(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_i(r)P_i \]

- \( V_W(r) \) Wigner potential (depends only on \( r \))
- \( V_r(r) \) spin
- \( V_\sigma(r) \) parity
- \( V_i(r) \) isospin
Potential between 2 nucleons

Spin and isospin dependence of the nucleon-nucleon interaction can be described in terms of projection operators.

Def. Spin exchange operator

\[
P^\sigma = \frac{1 + (\mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2)}{2}
\]

interchanges the spin variables of the pair \( P^\sigma \chi(\mathbf{\sigma}_1, \mathbf{\sigma}_2) = \chi(\mathbf{\sigma}_2, \mathbf{\sigma}_1) \)

It can be shown that:

\[
P^\sigma |\psi\rangle = \pm 1 |\psi\rangle \quad +1 \quad (S=1)
-1 \quad (S=0)
\]

For spin:

\[
|S = 1, m_s = +1\rangle = |\uparrow \uparrow\rangle
|S = 1, m_s = 0\rangle = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle + |\downarrow \uparrow\rangle)
|S = 1, m_s = -1\rangle = |\downarrow \downarrow\rangle
|S = 0, m_s = 0\rangle = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)
\]

The spin-spin part of the interaction potential can be written as

\[
V_\sigma(r) \frac{1}{2} (1 + \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2) \equiv V_\sigma(r)P^\sigma
\]
Potential between 2 nucleons

The central part of nucleon-nucleon potential

\[ V_C(r) = V_W(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_i(r)P_i \]

- **Wigner potential** (depends only on \( r \))
- **parity**
- **spin**
- **isospin**

\[ V_W(r) \]
Potential between 2 nucleons

Parity dependence

$P_r$ - parity of the wavefunction that describes the two particles (exchanges the space coordinates of two particles $P_r \psi(r_1, r_2) = \psi(r_2, r_1)$

$P_r |\psi\rangle = \pm 1 |\psi\rangle$  $P_r = +1$ (-1) if the wavefunction is even (odd)

For two particle system in a state of definite orbital angular mom. $P_r = (-1)^l$

Majorana potential: $V_r(r) P_r$
Potential between 2 nucleons

The central part of nucleon-nucleon potential

\[ V_C(r) = V_W(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_i(r)P_i \]

- \( V_W(r) \) Wigner potential (depends only on \( r \))
- \( V_r(r) \) spin
- \( V_\sigma(r) \) parity
- \( V_i(r) \) isospin
Potential between 2 nucleons

Isospin dependence

\( P_i \) - isospin operator, which changes the isospin of the two particles.

\[ P_i = -P_\sigma P_r = -P_H \quad P_H \quad \text{ - the Heisenberg operator} \]

The isospin part of the interaction potential can be written as

\[ V_i(r) \frac{1}{2} (1 + i_1 \cdot i_2) \equiv V_i(r)P_i \]
The two-body nuclear potential - Summary

The most general form of the nuclear potential must be a function of \( \sigma_1, \sigma_2, i_1, i_2, r, p \) (spins, isospins, relative position, relative momentum) and have a form:

\[
V(r; \sigma_1, \sigma_2, i_1, i_2) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_i(r)(i_1 \cdot i_2) + V_{\sigma i}(r)(\sigma_1 \cdot \sigma_2)(i_1 \cdot i_2) + V_{LS}(r)(L \cdot S) + V_{LSi}(r)(L \cdot S)(i_1 \cdot i_2)
\]

Central force

\[ + V_T(r)S_{12} + V_{Ti}(r)S_{12}(i_1 \cdot i_2) \]

Spin-orbit

\[ + V_Q(r)Q_{12} + V_{Qi}(r)Q_{12}(i_1 \cdot i_2) \]

Tensor force

\[ + V_{pp}(r)(\sigma_1 \cdot p)(\sigma_2 \cdot p) + V_{ppi}(r)(\sigma_1 \cdot p)(\sigma_2 \cdot p)(i_1 \cdot i_2) \]

12 unknown functions, use nucleon-nucleon scattering to find out about the nuclear force
Nucleon-Nucleon Potential

- Bound states (e.g. deuteron)
- Nucleon-nucleon scattering (low energies)

Nucleon-nucleon scattering at low energies can be described by non-relativistic Quantum Mechanics. Nucleons are assumed to be point-like objects with spin and isospin. Experiments thus must involve polarized nucleons (both proton and neutron).

**Scattering data:**
- it is common to reduce the experimental information to phase shifts $\delta_l$ for different $l$-partial waves.

**Method:**
- compare calculated phase shifts and those extracted from experimental data.