Lecture 12:

3. Properties of Nuclei

- Lepton-nuclei scattering, the EMC effect
- Deuteron - properties

Textbook (S. Wong, chapter 3; Povh 2.4, 6.2, 7.4)
Supplemental: D. Griffiths, Introduction to Elementary Particles on QM (Glebsh-Gordan coefficients)

03/27 (Friday) Quiz (Lecture 12 and 13)
Deep Inelastic Scattering from **Nuclear Targets**
(unpolarized)

- **Differential cross section** is described by two nuclear structure functions:
  \[
  F_{1,2}^A \equiv F_{1,2}^A(x_A, Q^2) \\
  x_A = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_A \gamma} = x \frac{M}{M_A} \\
  0 \leq x_A \leq 1 \\
  0 \leq x \leq \frac{M_A}{M} \approx A
  \]

- Differential cross section per nucleon (convenient to use structure functions which depend on the \(x\) Bjorken for a free nucleon)

  \[
  \frac{d^2 \sigma^A}{dx dQ^2} = \frac{4\pi \alpha^2}{Q^4 x} \left[ x y^2 F_1^A(x, Q^2) + \left(1 - y - \frac{\gamma^2 y^2}{4}\right) F_2^A(x, Q^2) \right] \\
  y = \frac{\nu}{E} \\
  \gamma = \frac{2M_x}{\sqrt{Q^2}} \rightarrow \text{small}
  \]

Recall: unpol. cross section for DIS of lepton off nucleon (proton)

\[
\frac{d^2 \sigma_{\text{unpol}}}{dx dQ^2} = \frac{4\pi \alpha^2}{Q^4 x} \left[ x y^2 F_1(x, Q^2) + \left(1 - y - \frac{\gamma^2 y^2}{4}\right) F_2(x, Q^2) \right]
\]
Deep Inelastic Scattering from Nuclear Targets

No much interest before 1982: *Incoherent scattering* from $A$ nucleons

\[ A \cdot F_2^A \approx Z \cdot F_2^p + (A - Z) \cdot F_2^n \]

For isoscalar targets \[ (A - Z) = Z = A/2 \]

\[ F_2^A \approx \frac{F_2^p + F_2^n}{2} \]

Notation (this lecture): $F_2^A$ (*per nucleon*)
EMC (European Muon Collaboration) at CERN:

- ~10 times *higher* beam energy than earlier DIS experiments,
- An iron target to boost luminosity...
- (Fe: A=56, Z=26; Z>A-Z)

**DIS - Surprises with Nuclei**

**QPM interpretation:**
valence quarks in nucleus carry less momentum than in nucleon
Structure function per nucleon of a nucleus with mass number $A$

$F_2^A$

Structure function per nucleon of a deuteron, weakly bound state of $p$ and $n$

$F_2^d \approx \frac{F_2^p + F_2^n}{2} = F_2^N$

~ isospin averaged nucleon structure function

$F_2^A / F_2^d \rightarrow 1$

In the absence of nuclear effects the ratio is normalized to 1

Nuclear effects observed in scattering of $\gamma^*$ (DIS) from nuclei
Nuclear EMC Effect

\[ F_2^A \]

\[ F_2^d \approx \frac{F_2^p + F_2^n}{2} = F_2^N \]

\[ \frac{F_2^A}{F_2^d} \rightarrow 1 \]

Structure function per nucleon of a nucleus with mass number A

Structure function per nucleon of a deuteron, weakly bound state of p and n, ~ isospin averaged nucleon structure function

In the absence of nuclear effects the ratio is normalized to 1

Nuclear effects observed in scattering of \( \gamma^* \) (DIS) from nuclei
Nuclear EMC Effect

Each of these regions has sensitivities to different many-body effects.

Nuclear effects observed in scattering of $\gamma^*$ (DIS) from nuclei.
Nuclear EMC Effect

- shadowing region: $x < 0.1$

Interpretation: Coupling of the photon ($qq\bar{q}$) to the strongly interacting quarks or to the overlap in the longitudinal direction of small-$x$ partons from different nuclei.

Homework, Sec. 7.4
Nuclear EMC Effect

- Shadowing (low x) effect increases with nuclear mass number, for example: \( x \approx 0.01 \) \( \frac{F_2^A}{F_2^d} \sim A^{\alpha-1} \)
- Weak Q^2 dependence
Nuclear EMC Effect

- **Shadowing (low x)**
  - shadowing region: \( x < 0.1 \)
  - Anti-shadowing (enhancement region): \( 0.1 < x < 0.2 \)

*Interpretation: coherent interaction with more than one nucleon*
Nuclear EMC Effect

- Shadowing region: $x < 0.1$
- Anti-shadowing (enhancement region): $0.1 < x < 0.2$
- Region of the “EMC effect”: $0.2 < x < 0.8$

Interpretation: Valence quarks carry less momentum in a bound nucleon. Momentum distribution of quarks is shifted toward smaller $x$ in a nucleus.
Nuclear EMC Effect

- **Shadowing region:** $x < 0.1$
- **Anti-shadowing (enhancement region):** $0.1 < x < 0.2$
- **Region of the “EMC effect”:** $0.2 < x < 0.8$
- **Fermi motion region:** $x > 0.8$
Nuclear EMC Effect

"EMC means Everyone’s Model is Cool"

Numerous models to explain the effect, based on:

- interactions between quarks in different nucleons
- "swelling" of the nucleon radius within nucleus
- coalescence (merging) of nucleons to form multiquark clusters
- kinematic effects due to nuclear binding (reduction in the effective mass)
- correlation between nucleons
- Fermi motion

Textbook effect, remains in search of comprehensive explanation: key for understanding the nuclear force on the basis of fundamental interactions between quarks and gluons.

See e.g. M. Arneodo, Phys. Rep. 240 (1994) 301
Nuclear EMC Effect

New data changed the experimental conclusion that the EMC effect follows the average nuclear density and instead suggested that the effect follows local nuclear density.

Correlations lead to local densities in the nucleus that are several times as high as the average nuclear density of 0.17 GeV/fm$^3$.

http://cerncourier.com/cws/article/cern/53091
http://cerncourier.com/cws/article/cern/37330
**EMC Effect in the Deuteron**

**Deuteron** – the simplest nucleus, weekly bound state of p and n

\[ 2F_2^d \neq F_2^p + F_2^n = 2F_2^N \]

**Nuclear effects** present: Fermi motion, shadowing, EMC effect
Nuclear Force:

- short range force between colorless nucleons
  "short" means the order of the nucleon diameter
- responsible for holding the nucleus together

We can study the nuclear force (or equivalently the nuclear potential) by analyzing two-body systems:

- the proton-neutron bound state (the deuteron)
- nucleon-nucleon scattering at low energies (elastic scattering)

Proton-proton scattering cross section

It was found that the nuclear force depends upon the total spin and isospin of the two nucleons.
The Deuteron

- nucleus of deuterium, bound state of proton and neutron (the simplest nucleus)
- isotope of hydrogen
- it exists in its ground state only and has no excited states. Thus when energy is supplied to it, it readily disintegrates instead of going into higher energy states.

Properties:

- Binding energy $\Delta E=2.2$ MeV

$$M_d = 1.8756 \text{ GeV}$$
$$M_p + M_n = 0.9383 \text{ GeV} + 0.9396 \text{ GeV} = 1.8779 \text{ GeV}$$

Question: Why deuteron stable? i.e. why neutron inside the deuteron doesn’t undergo $\beta$ decays?
Isospin

Isospin (I) - a quantum number related to the strong interaction

Introduced by W. Heisenberg in 1932 to explain symmetries of the then newly discovered neutron:

- proton mass ~ neutron mass, the only difference is charge
- strength of the strong interaction between 2 nucleons is the same.

Proton and neutron = different states of the same particle (nucleon). Since a spin $\frac{1}{2}$ particle has 2 states, the two were said to be of isospin $I=\frac{1}{2}$.

Isospin dublet ($l=1/2$)

- proton $|p\rangle = |I = \frac{1}{2}, I_3 = +\frac{1}{2} \rangle$
- neutron $|n\rangle = |I = \frac{1}{2}, I_3 = -\frac{1}{2} \rangle$

Adding isospins: the same rules as for angular momenta
Reminder

Quantum Mechanics: adding angular momenta

\[ \vec{J} = \vec{J}^1 + \vec{J}^2 \]

\[ |j, m\rangle = |j_1, m_1\rangle, |j_2, m_2\rangle \quad m = m_1 + m_2 \]

\[ j = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots, (j_1 + j_2) - 1, (j_1 + j_2) \]

Example: Adding 2 spins \( \frac{1}{2} \) (the same for isospins)
Reminder

Quantum Mechanics: adding angular momenta

\[ \vec{J} = \vec{J}^1 + \vec{J}^2 \]

\[ |j,m\rangle: |j_1,m_1\rangle, |j_2,m_2\rangle \quad m = m_1 + m_2 \]

\[ j = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots, (j_1 + j_2) - 1, (j_1 + j_2) \]

Example: Adding 2 spins \( \frac{1}{2} \) (the same for isospins)

\[ \vec{J}_1 = \vec{S}_1 \quad \vec{J}_2 = \vec{S}_2 \quad \vec{J} = \vec{S} = \vec{S}_1 + \vec{S}_2 \]

\[ j_1 = s_1 = \frac{1}{2} \quad j_2 = s_2 = \frac{1}{2} \]

\[ j = s = |s_1 - s_2|, \ldots, (s_1 + s_2) \]

\[ s = \left| \frac{1}{2} - \frac{1}{2} \right|, \ldots, \left( \frac{1}{2} + \frac{1}{2} \right) \Rightarrow s = 0, 1 \]

Result:

\[ |s,m_s\rangle \quad \text{where} \quad m_s = -s, -s+1, \ldots, s-1, s \]

\[ s = 0 \Rightarrow m_s = 0 \]

\[ s = 1 \Rightarrow m_s = -1, 0, 1 \]

4 states
The Deuteron, Properties

Add 2 nucleons together:

\[ p = |\uparrow\uparrow\rangle = \left| I = \frac{1}{2}, I_3 = +\frac{1}{2}\rightangle \quad n = |\downarrow\downarrow\rangle = \left| I = \frac{1}{2}, I_3 = -\frac{1}{2}\rightangle \]

- **Isotriplet:** \( I = 1, \quad I_3 = -1, 0, 1 \)

\[ |I = 1, I_3 = +1\rangle = |\uparrow\uparrow\rangle = p(1)p(2) \]

\[ |I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}} (p(1)n(2) + n(1)p(2)) \]

\[ |I = 1, I_3 = -1\rangle = |\downarrow\downarrow\rangle = n(1)n(2) \]

Reminder (Quantum Mechanics)

![Notation: J J ... M M ... Coefficients](image)

1/2 x 1/2

![Coefficients](image)

the Clebsch-Gordan coefficients
The Deuteron, Properties

- Isospin = 0 for the deuteron

Add 2 nucleons together:

\[
p = |\uparrow\rangle = \left| I = \frac{1}{2}, I_3 = +\frac{1}{2} \right> \quad n = |\downarrow\rangle = \left| I = \frac{1}{2}, I_3 = -\frac{1}{2} \right>
\]

- **isotriplet:** \( I = 1, I_3 = -1, 0, 1 \)

\[
| I = 1, I_3 = +1 \rangle = |\uparrow\uparrow\rangle = p(1)p(2)
\]

\[
| I = 1, I_3 = 0 \rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}} (p(1)n(2) + n(1)p(2))
\]

\[
| I = 1, I_3 = -1 \rangle = |\downarrow\downarrow\rangle = n(1)n(2)
\]

- **isosinglet:** \( I = 0, I_3 = 0 \)

\[
| I = 0, I_3 = 0 \rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}} (p(1)n(2) - n(1)p(2))
\]

**deuteron**

Experimentally there are no bound states of two protons or two neutrons, thus deuteron must be isosinglet (evidence for strong attraction in the I=0 channel but not in the I=1 state)
See you on Wednesday

Questions, comments:
e-mail: Joanna.Kiryluk (at) stonybrook.edu

Or

C109 (Physics Building)