PHYSICS 252   EXPERIMENT NO. 6
SCATTERING

Introduction

The purpose of this laboratory is to show how certain characteristics of an object can be determined from the distribution of particles (or radiation) scattered from the object. In particular, we will use the scattering of a beam of laser light to indirectly measure the radius of several plexiglass cylinders. The method is similar in principle to that used in particle beam experiments to determine the size and shape of target nuclei.

Measurement Method

Imagine a well-collimated beam of light striking a polished circular cylinder as shown in the Figure. \( R \) is the cylinder radius, and \( b \) is the "impact parameter", i.e. the distance between beam and the parallel line passing through the center of the target. After striking the target, the beam is reflected such that the angle of reflection, \( \alpha \), equals the angle of incidence. From geometry it follows that \( \sin \alpha = b/R \). Defining the scattering angle \( \theta = \pi - 2\alpha \), we find that \( \alpha = (\pi - \theta)/2 \). Thus:

\[
\sin \alpha = \sin[(\pi - \theta)/2] = \cos[\theta/2] = b/R , \quad \text{from which} \quad b = R \cos[\theta/2] 
\]

Suppose \( b \) is changed by a small amount \( db \) (infinitesimal notation). This will cause a corresponding change \( d\theta \) in the scattering angle \( \theta \) given by

\[
db = -(R/2) \sin[\theta/2] \ d\theta . \tag{1}
\]

If this experiment is performed \( dN \) times, while keeping the laser beam in the interval (\( b, b+db \)), one will measure \( dN \) scattering "events" in the angular interval (\( \theta, \theta+d\theta \)). Thus the number of scattering events per unit angle is then

\[
dN/d\theta = (dN/db) \ (R/2) \sin[\theta/2] , \tag{2}
\]

where we used equation (1) and dropped the minus sign as we are interested only in the absolute values of the quantities concerned, i.e. the scattering rate. \( dN/db \) can be regarded as the incident intensity. If, for example, one were to perform four scattering measurement for each 0.1 cm step in \( b \), then \( dN/db = 4/0.1 = 40 \) events per centimeter. Experimentally, \( dN/d\theta \) is approximated by marking off the chamber wall in \( 10^\circ \) increments, and then counting how many scattering events end up in each angular interval, or "bin", as \( b \) is varied. Thus \( dN/d\theta \equiv \Delta N/\Delta \theta \), where \( \theta \) is expressed in radians when using equation (2).
Analysis

For each of three cylinders, plot $dN/d\theta$ on the $y$-axis and $\sin\theta/2$ along $x$. If equation (2) holds, you should find a linear relationship, with slope equal to $(dN/db)(R/2)$, from which you can determine $R$. Note that the angular scale on the chamber wall actually measures $\phi$, not $\theta$ (see Figure)! For $R$ small compared to the chamber radius, $\theta \approx \phi$, but otherwise you will have to find a way to calculate $\theta$ from $\phi$ (such a derivation will need knowledge of both $R$ and chamber radius $r$, which may seem like "cheating", but it can be done iteratively; also, in actual scattering experiments the nuclear diameter is negligibly small compared to the size of the detector!). For each cylinder compare the derived value of $R$ (with its calculated errors!) with the actual radius of the cylinder.

Revised: August 14, 1998